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ARC SPECTRA.

By Arthur L. Foley.

WHEN the image of a normal electric arc is examined it is seen, as regards color, to consist of three regions. The central portion, 1, Fig. 1, is violet. At times it extends entirely across from the positive to the negative carbon, as in Fig. 1. At other times it appears at the positive carbon only, as in Fig. 2. It is always broadest at the positive carbon.

Surrounding the violet center is a sheath of blue (2), which frequently extends entirely across the arc at the negative carbon, as in Fig. 2, but never at the positive carbon. The outside sheath (3) is yellow. It is narrow or thin at the lower carbon and broad at the upper carbon, the lower end of which it completely surrounds.

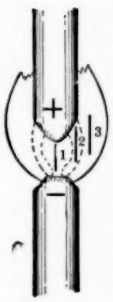

Fig. 1.

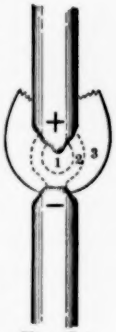

Fig. 2.

A reversal of the direction of the currents shows that sheaths (1) and (2) are quite definite in position as regards the positive and negative carbons. The flame like character of (3) is shown by the fact that it is practically independent of the direction of the current.

These different sheaths, which present such a contrast in color, were examined spectroscopically by Lockyer[1] in 1879. Prior to that time Lockyer and others had worked with the spectrum of the arc maintained by a current from a battery. To minimize trouble and expense, it was customary

[1] Proc. Roy. Soc., XVIII., p. 425, 1879.

to reduce the number of battery cells required by burning a short arc. Indeed the image of the arc was so short that in order to obtain lines of any length it was necessary to focus the image of a horizontal arc upon the vertical slit of a spectroscope. Under such conditions the regions before mentioned scarcely appear.

When Lockyer was enabled to substitute a dynamo for his battery, he began to use a long vertical arc and a vertical slit. He focussed different parts of the arc upon the slit of a spectroscope, and obtained photographs of the spectra. He remarked upon the theoretical importance of the phenomena observed, and upon the great complexity of the various flame sheaths, especially when certain foreign substances were introduced into the arc. His results are interesting, but not very definite. He found that, in the ordinary arc, the carbon flutings clung to one pole and the calcium lines to the other. But when calcium was introduced into the arc, "the blue calcium lines were visible alone at one pole and the H and K lines, without the blue, at the other." He found a progression of lines from pole to pole. "They lie en échelon along the spectrum." Some lines, while extending entirely across the spectrum, were irregularly thickened at different levels of the arc. A somewhat analogous result has been found in the spectra of gas flames at different levels.[1]

Miss C. W. Baldwin[2] recently investigated the subject with somewhat surprising results. She found that the carbon and many of the metallic lines were absent or greatly weakened in the outer sheaths, *while certain other lines, those most brilliant near the negative carbon, were strengthened.* The introduction of potassium, sodium, lithium, barium, calcium, and strontium changed the characteristic band spectrum of the arc by weakening those lines which were strongest at the positive carbon, while copper and silver had but little effect upon the original spectrum. Zinc and cadmium appeared to affect most the lines near the negative carbon.

The present research was undertaken for the purpose of verifying

[1] The Spectra of Metals at the Base of Flames. M. Gouy, Phil. Mag., Vol. III., p. 238, 1879.

[2] A Photographic Study of Arc Spectra. C. W. Baldwin, PHYSICAL REVIEW, Vol. III., pp. 370 and 448, 1896.

the results of previous investigations on this subject, and of so extending the work as to make it possible to arrive at some more definite conclusion.

Apparatus.

A Rowland concave grating of 6½ feet radius and 14,438 lines to the inch, was placed upon a Brashear mounting, Fig. 3. The grating was the choice of three, selected because it gave a uniformly bright first spectrum, and very weak spectra of higher orders.

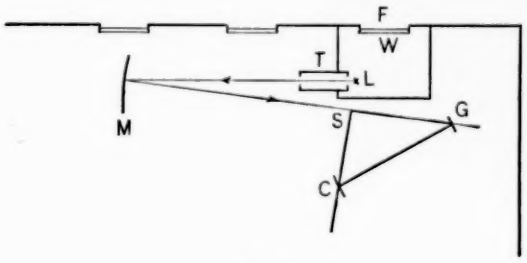

Fig. 3.

The slit *S* had the usual adjustments for width, length, and direction. The width generally used was 0.02 mm. The light from an arc lamp *L* was focussed upon the slit by a concave mirror *M*, of ten inch aperture and twelve feet radius. The mirror, being somewhat scratched and tarnished, was resilvered by the Brashear process as described by Wadsworth.[1] The lamp, which was in the same horizontal plane with the mirror, slit and grating, was placed inside a small closet built out in the room, and as near the slit as possible. The image was the same size as the arc, and as it lay near the axis of the mirror, the distortion was reduced to a minimum. A tube *T*, with a diaphragm at each end, served to cut off superfluous light. However, it was found impracticable to screen off all of it. The apparatus was set up in a large room with a number of windows *W* and other openings. It was decided to enclose the slit, grating, and camera, and to make no attempt to darken the room.

The first arrangement consisted of a number of metal rings placed

[1]Astrophysical Journal, I., p. 252, 1895.

crosswise at intervals along the track *SG* and the carriage arm *CG*. Those on *SG* were arranged to slide along the track so as to permit of the carriage arm being moved into any position desired. About these rings was wrapped black cotton flannel, in such a way as to form two hollow cylinders completely enclosing the camera and grating and all of *S* but the slit itself. This arrangement was not satisfactory. When photographing the red end of the spectrum, which required a comparatively long exposure, the violet portion of the first spectrum together with the spectra of higher orders, fell upon the walls of the tube between the grating and camera, and although the nap of the cloth was roughed up and turned inside, it was found that there was sufficient reflected light to fog the plate. The arrangement was modified by removing the cloth and rings along *CG*, and substituting for the cloth cylinder a long tapering box, Fig. 4, somewhat larger than the camera box at the end *C*, and about the size of the grating holder at the end *G*. A portion of the side of the box at the end *G* and next *SG* was moved to allow the light from the slit to reach the grating. Along the inside walls of the box were tacked strips or vanes of tin in the manner shown in the figure, and the whole interior was painted dead black. A piece of cloth at each end served to make the apparatus light tight. This arrangement was entirely satisfactory. An exposure of thirty minutes failed to show any fogging of the plate due to light reflected from the walls of the box, or to the light of the room. To move the carriage arm into a new position it was necessary only to adjust the length of the cloth tube along *SG*.

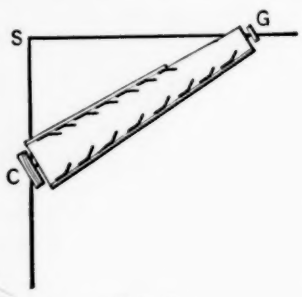

Fig. 4.

The camera box provided with the mounting was arranged for 1½ x 5 inch straight glass plates. This box was discarded, and one was constructed for 2 x 8 inch plates, curved to 3¼ feet radius to conform to the grating. Celluloid films were tried, but they were found to be unsatisfactory where accurate measurements were necessary. The thinnest glass plates that could be obtained (three

dry plate companies were tried) invariably broke when an attempt was made to bend them to the proper curvature. It was noticed that they did not break when put in backwards; that is, with the film on the back or convex side. This suggested the plan of coating both sides, or of pasting on the back of the plate something that would act the part of the film. Rice paper was tried without success. The paper proved to be less tenacious than the film. The whole difficulty was overcome by changing the radius of curvature of the plate holder from 3¼ to 6½ feet. The plates were easily bent to the latter curvature without breaking. That the spectrum remained a normal one is shown as follows:

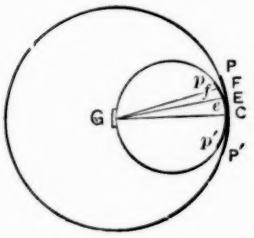

Fig. 5.

Let G, Fig. 5, be a concave grating of radius GC. With GC as diameter construct a circle. With G as center and GC as radius construct a second circle. The two circles are tangent at C.

All the spectra from the grating (supposed mounted as before described) are in focus on the first circle; and the usual method is to bend the plate pCp', into an arc of this circle. But suppose the plate PCP' lies in the circumference of the second circle, and consider only the rays of light that proceed from the center of the grating, as GC, GE, etc. As the arc Ce measures the angle GCe at the circumference of the first circle, and the arc CE measures the same angle at the center of the second, the arc Ce=the arc CE. Likewise $fe=FE$, so that equal lengths on the plate PCP' represent equal differences in wave-length, as they do on the plate pCp'. The plate PCP' is not accurately in focus except at C; but unless one uses longer plates than is usual in such work, the definition is practically uniform. The lines at the ends of the eight-inch plates used in this investigation appeared just as sharp and clear under the microscope, as did those at the center. Even if sufficiently thin glass plates can be secured, the fact that such plates are seldom plane, and that they warp, and bend very unevenly, suggests the advisability of reducing the curvature of the plate when a short focus grating is used.

I. *The Spectrum of the Ordinary Arc.*

This study was made under four conditions: (1) a vertical arc and a vertical slit, (2) a vertical arc blown out by a horseshoe magnet, (3) a horizontal arc and a vertical slit, (4) a horizontal arc and a horizontal slit.

1. *Vertical arc and vertical slit.*—By means of a rack attached to the upper carbon holder of a Thomson-Houston arc lamp, and a pinion which passed through the wall of the closet and was operated by a knob near the slit, the length of the arc was kept equal to or greater than the length of the slit—about 1.5 cm. When the arc happened to break during an exposure the slit was covered until the carbons were brought together and the arc drawn out as before. In this way photographs were obtained free from the continuous spectrum of the hot carbon points.

Photographs were taken with the slit in a dozen or more positions. Tables I, II and III give the results of a study of the three regions of the arc with the slit vertical and through the center of each region. Two exposures were made in each position, three seconds and thirty seconds in the violet and blue regions, ten seconds and thirty seconds in the outer or yellow sheath. An exposure of three seconds in the latter brought out only ten lines.

The tables extended from $\varphi = 3092$ to $\varphi = 5015$. The measurements were made with the plate on the bed of a dividing engine, and the lines were identified by reference to the tables of Kayser und Runge,[1] Rowland,[2] and Hasselberg.[3]

On each plate there were traces of lines too hazy or indefinite to be accurately measured. A few lines on Plate 2 were so blurred by over-exposure that their measurements were doubtful. With these exceptions every line on each of the six plates was carefully measured. The accuracy of the measurements was sufficient for the identification of the lines in almost every case. The intensity of each line was estimated at three points: at the upper end or near the upper or positive carbon, at the center, and near the lower end or negative carbon. In Table I the lines are classified as to

[1] Kayser und Runge, Ueber die Spectren der Elemente.
[2] Astrophysical Journal, I., p. 29, 1895.
[3] Astrophysical Journal, IV., p. 116, 1896.

TABLE I.

Elements.	Region 1. Central Violet.		Region 2. Blue.		Region 3. Outer Yellow.	
	Plate 1. Exposure 3 Seconds.	Plate 2. Exposure 30 Seconds.	Plate 3. 3 Seconds.	Plate 4. 30 Seconds.	Plate 5. 10 Seconds.	Plate 6. 30 Seconds.
C	313	450	60	274	1	8
Fe	84	107	29	91	47	74
Ca	14	17	3	15	8	15
Ti	0	20	0	1	0	1
K	3	4	2	3	2	3
Al	2	3	2	2	2	2
Mn	2	2	0	2	1	1
Cu	0	2	0	0	0	0
Ba	1	2	0	1	1	1
Na	1	1	0	1	0	1
Si	1	1	1	1	0	1
Not identified	4	50	0	1	0	0
Faint or blurred . .	29	37	38	29	6	13
To. No. lines identified .	421	709	97	391	62	107
To. No. lines visible . .	454	796	135	421	68	120

Whole number of lines visible, 1994.
Whole number of lines measured, 1842.
Whole number of lines identified, 1787.

number and source. The table shows that the lines rapidly decrease in number as the slit is moved from the center of the arc to the outer edge. This is due chiefly to the fading out of the carbon bands, only a few of the stronger carbon heads being visible in the outer sheath. But the fading out is not confined to the carbon, as the lines of all the other elements of the table show the same tendency, though to a much smaller degree. This fact is better brought out in Table II., where the general character of the lines is indicated. The estimates given for any element represent as nearly as possible the average of all the lines of this element, in the given position. The first estimate is for the upper end of the line, near the positive carbon; the second for the center; the third for the lower end, near the negative carbon. There is a decrease not only in the *number* of

TABLE II.

Element.	Region 1. Central Violet.	Region 2. Blue.	Region 3. Outer Yellow.
	Strong	Weak	Trace
C	Fairly strong	Trace	Absent
	Weak	Absent	Absent
	Strong	Fairly strong	Weak
Fe	Strong	Fairly strong	Fairly strong
	Strong	Fairly strong	Weak
	Strong	Strong	Fairly strong
Ca	Strong	Strong	Strong
	Very strong	Very strong	Fairly strong
	Absent	Absent	Absent
Ti	Very Weak	Trace	Trace
	Absent	Absent	Absent
	Weak	Weak	Very weak
K	Weak	Weak	Very weak
	Strong	Fairly strong	Weak
	Fairly strong	Weak	Weak
Al	Strong	Fairly strong	Weak
	Strong	Fairly strong	Weak
	Weak	Very weak	Trace
Mn	Weak	Very weak	Absent
	Very weak	Very weak	Absent
	Very weak	Absent	Absent
Cu	Very weak	Absent	Absent
	Very weak	Absent	Absent
	Fairly strong	Fairly strong	Weak
Ba	Fairly strong	Fairly strong	Weak
	Strong	Fairly strong	Weak
	Fairly strong	Absent	Absent
Na	Fairly strong	Trace	Trace
	Strong	Trace	Trace
	Strong	Fairly strong	Trace
Si	Fairly strong	Weak	Absent
	Weak	Trace	Trace

lines as the slit is moved outwards from the center of the arc, there is a corresponding decrease in the average *intensity*. The two tables are in complete agreement, for the intensities would be expected to decrease as the number of lines diminishes.

On pages 376 and 378 of Miss Baldwin's paper, previously referred to, a number of lines are given with the statement that they appear to be stronger in the outer regions of the arc than in the central region. To test this matter Plates 2, 4, and 6, which were exposed equal times (30 seconds) and developed under the same conditions, were placed side by side on the carriage bed of a dividing engine. They were adjusted so that the microscope, sliding in its track at right angles to the bed of the engine, could be brought successively over the same line on each. Any other line could then be brought under the microscope by moving the carriage.

TABLE III.

Wave-length.		Region 1. Central Violet.	Region 2. Blue sheath.	Region 3. Outer Yellow.
3624.15	Ca			
3630.82	Ca	Weak		
3644.45	Ca	Weak		
3706.18	Ca	Weak	Weak	Very weak
3737.08	Ca	Fairly strong	Fairly strong	Fairly strong
3812.20	C	Fairly strong	Weak	
3816.02	Fe	Fairly strong	Fairly strong	Weak
3820.59	Fe-C	Strong	Strong	Fairly strong
3827.04	Fe			
3832.46	Mg			
3834.42	Fe	Fairly strong	Fairly strong	Fairly strong
3838.44	Mg			
3841.24	Fe-Mn	Fairly strong	Weak	Weak
3856.54	Fe	Fairly strong	Fairly strong	Fairly strong
3860.05	Fe-C	Strong	Fairly strong	Fairly strong
3933.83	Ca	Very strong	Very strong	Strong
3960.05				
3968.63	Ca	Very strong	Very strong	Strong
4076.96	Fe			
4226.91	Ca	Very strong	Very strong	Very strong
4299.14	Ca	Fairly strong	Fairly strong	Weak
4302.68	Ca	Strong	Fairly strong	Fairly strong
4318.80	Ca	Strong	Fairly strong	Fairly strong

This arrangement was advantageous, as a comparative study could be made of the same line on each of the three plates, at the same time. Table III gives the results of the study. A blank space indicates either that the line was absent, or that it was too hazy or too blurred to be measured.

The tables show that the yellow or flame portion of the arc was composed chiefly of the vapors of calcium and iron. The lines of these elements were relatively much stronger in the outer regions, than the lines of any other element. All the lines of any one element did not fade out at the same rate. Most of those of Table III were relatively much stronger in the outer regions of the arc than were the other lines due to the same element. But it does not appear that there are any lines which have a maximum of intensity in the outer sheaths. The opposite conclusion may have been brought about by failing to consider some of the following points:

a) Time of exposure. Table I shows that an exposure of three seconds in the violet region brings out almost four times as many lines as an exposure of thirty seconds in the yellow sheath, so one is inclined to expose a plate much longer in the outer region. Miss Baldwin[1] says that this was done in her investigation. If the absolute intensities of the lines are in question, the arc must be maintained as nearly constant as possible, and the exposures must be equally timed.

b) Time of development. When an exposure is made in the violet region, the carbon (or cyanogen) bands (heading near $\lambda = 3883$, $\lambda = 4216$, and $\lambda = 3590$) appear almost immediately after the plate is placed in the developer. Unless the exposure has been very brief a prolonged development causes the carbon lines near the several heads, especially on the side toward the shorter wave-lengths, to run together to such an extent that their identity is lost. So the tendency is to underdevelop. The reverse is the case in the outer sheaths of the arc where the carbon bands are weak or absent, and the metallic lines are not so numerous. When intensities are to be compared the plates must be developed under the same conditions. An underdevelopment frequently brings out differences of intensities that do not appear on a fully developed plate. The differences are

[1] PHYSICAL REVIEW, Vol. III., No. 17, p. 375.

usually better brought out in an underexposed plate which has been fully developed.

c) The tendency is to overestimate the intensities of the lines of the outer sheath, as they stand out prominently on a clear background, while many of the lines of the violet region are more or less masked by the superposed carbon spectrum.

d) Variations in the intensities of the lines due to accidental causes (fluctuations of the lamp, lack of uniformity of the carbons, etc.) must be eliminated by extending the study to a large number of plates.

In the present investigation a metronome was used to time the exposures. All the plates of any one series were placed in the same developing bath at the same time and were allowed to remain the same length of time. An amidol developer was used. Fifty-six plates were studied in this manner.

That there are no lines which have a maximum of intensity in the outer sheaths of the arc is not proven by the work already described. For, when the central or violet region was focussed on the slit, the latter received light from all the other sheaths also, and the spectrum obtained consisted of the superposed spectra of all the sheaths. In position (2) (Fig. 1) the slit received light from sheaths (2) and (3). In position (3) only was the light confined to a single sheath.

This objection loses a part of its force when one considers that a greater thickness of the outer sheaths was presented in positions (2) and (3), than in position (1), where the sheaths were normal to the slit, and that the light from the centra and hottest portion of the arc was somewhat diminished by the absorption of the outer and cooler portions.

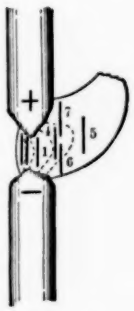

Fig. 6.

2. *Vertical arc blown out by magnet.*—When a vertical arc is blown sidewise by a horseshoe magnet, Fig. 6, the central region is but slightly displaced. The blue region is considerably thicker on the side opposite the magnet. The yellow region is almost entirely on that side. If now the slit is placed in position (1) it receives but little light from the thin outer sheaths.

Photographs were taken with the slit in the seven positions indi-

cated. No lines having maxima in the outer sheaths were found. If there were any such lines, it might be expected that an exposure in position (5) where there is a great mass and depth of flame would show stronger lines than an exposurein position (3). Such was not the case. There were lines at (3), also at (6) and (7), not found at (5).

When the slit was shortened until it extended but a fraction of the distance between the carbon points, lines were found at either (6) or (7) which were weak or absent at (1). When the slit was lengthened until it extended from pole to pole, no such lines were found. The explanation is not far to seek. The tendency of the carbon lines is to cling to the positive pole (Table II.) while most of the other lines are strongest at the negative pole. Consequently in position (7) we would expect some carbon lines not present at (5) or (6), and at (6) some metallic lines which are weak or absent at (5) and (7). A very short slit between the poles in position (1) does not give the true intensities of those lines which tend to cling to the carbons; many of these lines do not appear at all. But when the slit is lengthened until it extends from pole to pole, *all the lines of the arc are found to have a maximum of intensity in the central violet region.*

3. *Horizontal arc and vertical slit.*—When the arc was horizontal the atmospheric currents were so strong that it was not necessary to bring the magnet very near the arc to blow it out, as shown in Fig. 7.

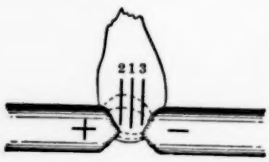

Fig. 7.

The current through the lamp was maintained at a much higher voltage than when the arc was vertical. Photographs were taken with the slit in three positions: (1) half-way between the carbons, (2) near the positive carbon, (3) near the negative carbon. As the slit extended almost across the arc the spectra of all the regions was obtained on each photograph, making a comparative study easy. All the lines were strongest in the violet region. The carbon lines were stronger in position (2) than in positions (1) or (3). The metallic lines averaged a trifle stronger near the negative carbon.

4. *Horizontal arc and horizontal slit.*—The arc was maintained as

before. The ends of the carriage beam of the Brashear mounting (Fig. 3), were firmly clamped to the track. The grating holder was modified so as to hold the grating in any position. The entire arrangement was then turned on its edge, so that the plane of the tracks was vertical, and the slit horizontal. Photographs were taken in three positions (Fig. 8). The results agreed with those obtained by the previous methods.

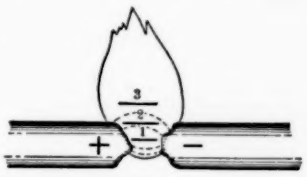

Fig. 8.

Neglecting for the present the thickening of the lines at the poles and considering each region of the arc as a whole, it may be concluded that the differences observed in the spectra are due chiefly, if not entirely, to temperature differences. This view was rejected by Miss Baldwin because the lines of those metals which have the highest melting points were not strongest in the hottest part of the arc. K, Na, and Li were strengthened at the negative carbon; so also were Ba, Sr, and Ca, which have relatively high melting points.

It will be shown in the second part of this paper that the strengthening of the metallic lines at the negative carbon is due to the electrolytic nature of the arc. Though the temperature of the negative carbon is considerably below that of the positive in the ordinary arc, usually the difference is not so great when a metallic salt is introduced into both carbons. Moreover, the temperature of the negative carbon is sufficiently high to vaporize rapidly the elements mentioned.

A careful examination of the image of the arc suggests that the colors of the three apparently well-defined regions are due to temperature differences. The violet center is surrounded by a blue sheath, but the change from the violet to the blue is gradual. The outer yellow sheath is green at its inner edge, and orange-red at the outer. In short, there is in a way a color progression, apparently due to a fall of temperature from the center of the arc outwards.

It was noted in the first part of this paper that the violet center sometimes extends to the negative carbon, sometimes not. It does so when the current is strong and the carbons are very hot. As the current is decreased the temperature of the carbons falls, and

the violet region contracts about the end of the positive pole, finally disappearing entirely.

II. *Spectra of the Elements.*

The spectra of twelve elements were studied to determine the nature of the lines near the carbons and directly between them. The spectrum was obtained by removing the core of *one* or *both* of the carbons and replacing it by the salt of the metal to be studied. The salts used were barium carbonate, sodium nitrate, the chlorides of zinc, calcium, strontium, potassium and lithium, the sulphates of chromium, cadmium and aluminum, and the oxides of rubidium and titanium.

Calcium and strontium were studied with both the vertical and the horizontal arc, under the four conditions described in part I of this paper. With the exception of the thickening of the lines at the poles, the differences in the spectra obtained from various parts of the arc could be attributed to temperature differences. Therefore, the study of the remaining elements was limited to the violet region of the vertical arc, with a vertical slit extending from pole to pole.

Six photographs were taken of the spectrum of each element. The conditions were as follows:

No. I., upper carbon plain (containing no metallic salt) and positive, salt in lower carbon.

No. II., upper carbon plain and negative, salt in lower carbon.

No. III., salt in upper negative carbon, lower carbon plain.

No. IV., salt in upper positive carbon, lower carbon plain.

No. V., salt in both carbons, upper carbon positive.

No. VI., salt in both carbons, lower carbon positive.

The accompanying plate gives five of the six photographs in the case of calcium chloride. The markings at the right end of each spectrum are intended to show at a glance the direction of the current and which of the carbons contained the salt.

The discussion of this part of the subject can be shortened by stating in advance some of the conclusions arrived at, as follows:

The arc is electrolytic. The electropositive elements seek the negative pole, and the electronegative the positive pole. Hence the thickening of the metallic lines at the negative carbon. Convection

currents due to heated gases in the arc may be sufficiently strong to mask the true nature of the lines.

No. V. (see plate), in which the core of each carbon is filled with calcium chloride, shows the thickening of the lines at the negative pole. The lines at the positive pole are not much stronger than in No. I., in which the lower carbon only contains the salt. In the latter the lines at the positive pole are due to the calcium vapors formed at the negative carbon, and carried upward by heat convection currents, for, in No. III., in which the negative carbon, still containing the salt, is placed above, almost all of the *calcium lines appear at the negative pole only.* The exceptions which may be seen in the photograph are due to the calcium contained as an impurity in the carbons. They were just as strong when both carbons were plain. No. IV., shows that convection currents alone do not bring about the conditions of No. III. In No. IV. the tendency of the lines to seek the negative carbon is sufficient to overcome the heat convection currents of the arc, and the lines are as strong at the lower carbon as at the upper.

All the lines given in Table IV. were visible on each of the negatives from which the accompanying plates were copied. Within the given limits the list embraces all but two of the lines catalogued by Kayser and Rungé. The absent ones are $\lambda = 4624.71$ and $\lambda = 4807.47$.

TABLE IV.

CALCIUM.

(Starred lines are present at both poles in Plate III, the others at the negative pole only.)

$\lambda = 3344.49$	$\lambda = 3949.09$	$\lambda = 4302.68$*	$\lambda = 4512.73$
3350.22	3957.23	4307.91*	4527.17
3361.92	3968.63*	4318.80*	4578.82
3468.68	3973.89	4355.41	4581.66
3474.98	4092.93	4425.61*	4586.12
3487.76	4095.25	4435.13*	4685.40
3624.15	4098.82	4435.86	4823.04
3630.82*	4226.91*	4454.97*	4833.85
3644.45	4240.58	4456.08*	4847.22
3706.18	4283.16*	4456.81	4878.34
3337.08	4289.51*	4508.04	5041.93
3933.83*	4299.14*	4509.89	

Of the forty-seven lines at the negative pole on plate No. III, only fourteen were visible at the positive carbon. They are starred in the table. As has been remarked before, these are the lines present in the arc spectrum between ordinary carbon points. Their intensity and number may be increased by increasing the time of exposure. But this increases the intensities of all the lines proportionately.

Ca photographs were chosen to illustrate the above effects because they represent about an average for the elements experimented upon. Ti did not show the effects at all. The lines extended across the spectrum with almost uniform intensity. Sometimes they were slightly intensified at the positive carbon, probably because of the higher temperature.

The Zn lines were slightly stronger at the negative carbon. The lines of Rb, K, Na, Li, Ba and Sr showed a much more decided preference for the negative pole than did the Ca lines. The preference was most marked in the case of Rb, though the most striking photographs were obtained with Ba, which has a large number of strong lines in this region of the spectrum.

As far as it could be determined the order of the elements, as regards the tendency of their spectral lines to cling to the negative carbon, is the same as their order in the electropositive-negative series. Table II. accords with this conclusion. The C, Si and Mn lines were found to be strongest at the positive pole, the Ca, K, Al, Ba and Na lines at the negative pole, while the Ti, Fe and Cu lines were about equally strong throughout.

The following experiment is more conclusive. The *negative* carbon of a horizontal arc was filled with calcium chloride. A *new plain* carbon formed the *positive* pole. A horizontal arc was used, otherwise the heat convection currents might have thrown doubt upon the results of the experiment. The slit was horizontal and extended from pole to pole. The carbons were placed 1 cm. apart and an arc was formed by passing between them a third carbon, which served to bring the poles in momentary contact. After one minute the current was shut off, and the carbon containing the Ca was replaced by a new plain carbon. The arc was then formed again by means of a third carbon, and the spectrum was photographed. *No Ca lines appeared except those always present in the ordinary arc.*

In the second part of the experiment a plain carbon was used as a *negative* pole, and the Ca was placed in the *positive* pole. The arc was formed as before, and allowed to continue for one minute. The positive pole containing the Ca was then replaced by a new plain carbon, the arc was formed and the spectrum photographed. *The Ca lines came out very clear and strong*, almost as strong as if the negative carbon had been *filled* with the salt. There can be but one conclusion. In the first case the Ca was in the negative pole and there was no tendency for it to pass over to the positive pole. In the second case it was placed in the positive pole, and it freely passed over to the plain negative pole. The latter became so impregnated with it that it was capable of giving a strong spectrum of Ca when it was afterwards used in an arc with a plain carbon.

Any of the elements, which show a marked tendency to cling to the negative pole, may be used instead of Ca in the above experiment. The result was doubtful in the case of Zn.

The electrolytic nature of the arc was further confirmed by a series of measurements of the voltage necessary to maintain an arc of given length between unlike electrodes. It was found that a much higher voltage was required when the metallic salt was placed in the negative pole than was necessary when the salt was in the positive pole. When one of the carbons was replaced by a metal rod, a higher voltage was required when the current was passing from the carbon to the metal, than when it was flowing in the opposite direction. The details of this portion of the work will be given in a subsequent paper.

III. *Spectrum of Carbon.*

A vast amount of work has been given to the investigation of the spectrum of carbon, and of carbon compounds. A greater amount remains to be done. The precise nature of the spectrum of carbon itself is still in question.

It has been noted before that the carbon or cyanogen bands, heading near $\lambda=3883$, $\lambda=4216$, and $\lambda=3590$, are the most prominent feature of the ordinary arc spectrum. The lines composing these bands are listed as carbon in Table I., though they are now usually attributed to cyanogen. They will be called the cyanogen

bands, while by the carbon bands will be meant those heading near $\lambda=4737$ and $\lambda=4382$. There are other carbon and cyanogen bands but they are not included in this investigation except when specifically mentioned.

Ångström and Thalèn[1] attributed the cyanogen bands to a compound of carbon and nitrogen. Lockyer[2] found them to be as strong when the arc between carbon poles was in dry chlorine as when in air. Consequently he pronounced them carbon bands. Liveing and Dewar[3] questioned the purity of the chlorine and carbon used by Lockyer. They arranged an arc inside a glass globe into which was passed continuously a stream of the gas under experiment. The cyanogen bands were weak or absent in chlorine, hydrogen, carbon dioxide and carbon monoxide; they were strong in air, nitrogen and ammonia. That cyanogen was formed by the arc in air was proved by analyzing the contents of the globe.[4]

Lockyer[5] carefully removed the air from a tube containing carbon tetrachloride. When a spark, without a condenser, was passed between platinum terminals sealed into the tube, the cyanogen flutings were quite strong. They disappeared when a condenser was used, but in neither case did any hydrogen or nitrogen lines appear. Liveing and Dewar[6] repeated Lockyer's work using greater care to obtain pure gases. They failed to find the cyanogen bands in carbon tetrachloride. They studied the spark in several other compounds of carbon. The cyanogen bands appeared only when the compound was one that contained nitrogen. They were subsequently able to reverse the bands.[7]

Kayser and Runge[8] formed an arc between carbon terminals inside a block of retort carbon. When a stream of CO_2 was passed through the enclosure the cyanogen bands were weakened and the

[1] "Nova Acta Reg. Soc. Upsal.," Ser. III, vol. IX.
[2] Proc. Roy. Soc., 27, p. 308, 1878.
[3] Proc. Roy. Soc., 30, p. 152, 1880.
[4] Proc. Soc., 30, p. 85, 1880.
[5] Proc. Roy. Soc., 30, p. 335, 1880.
[6] Proc. Roy. Soc., 30, p. 494 and 490, 1880.
[7] Proc. Roy. Soc., 33, p. 3, 1881.
[8] Ueber die Spectren der Elemente, Zweiter Abschnitt. Also Wied. Ann., 38, p. 80, 1889.

carbon bands strengthened. When air was introduced the former were strengthened and the latter weakened.

Crew and Basquin [1] obtained strong carbon and weak cyanogen bands with an enclosed arc between metallic electrodes in CO_2. When a small quantity of air was introduced along with the CO_2, the cyanogen bands were strengthened and the carbon bands weakened. Magnesium electrodes appeared to be more favorable to the production of the carbon spectrum than did electrodes of iron and some other metals. Liveing and Dewar [2] had previously obtained similar results with an arc between carbon poles in a magnesia crucible.

The following experiments were made by the writer with an enclosed arc lamp manufactured by the Helios Electric Company. Fig. 9 shows a sectional view of the lamp as modified for the purpose of this investigation. A glass globe (g) was covered by a metal cap (c) through which passed the upper carbon. The lower end of the globe was clamped between two asbestus-lined metal plates of appropriate shape. The lower carbon extended through the plates and the frame of the lamp. A solid upper carbon was used. The lower carbon, which was hollow, had several small holes (h) bored through its walls near the upper end, so that a gas blown in at the lower end would be introduced into both the arc and the globe. The lamp was not air-tight, but the circulation was very slow when the lower end of the hollow carbon was closed. When a gas was blown into the globe, the admission of air was prevented by the escaping gas.

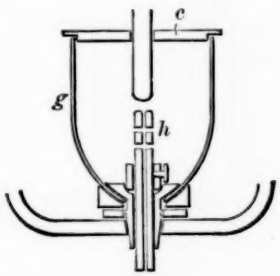

FIG. 9.

When experimenting with poisonous or offensive gases a fan motor (F, Fig. 3) was placed in the window of the closet to maintain a strong outward air current.

When the spectrum was photographed immediately after starting the arc, it was identical with the spectrum of the open arc. After a

[1] Astrophysical Journal, 2, p. 103, 1895.
[2] Proc. Roy. Soc., 34, p. 123, 1882.

minute or two the arc shortened and the metallic lines began to disappear. After ten minutes a hundred volts would maintain an arc scarcely 1 cm. long. *The metallic lines had disappeared almost completely.* The carbon and cyanogen bands were even stronger than in the first photograph. Air was then passed through the hollow carbon into the arc and globe. The arc lengthened and the metallic lines reappeared. When the current of air was shut off and a stream of CO_2 turned on, the metallic lines were weakened, but they did not disappear as long as the CO_2 was flowing.

It appears that a comparatively rapid disintegration of the carbon poles is necessary to furnish enough material in the arc to bring out clearly the metallic lines, which are due to very small quantities of the metals contained in the carbons as impurities. The rapidity of the disintegration depends upon the amount of O present in the globe. After the O had become exhausted by allowing the arc to burn a few minutes the wasting away of the poles was very slow, indeed. It was not more than 0.1 cm. per hour for the positive carbon, and very much less for the negative. When CO_2 was introduced the wasting of the poles increased, and, as noted, before the metallic lines appeared. Air still further increased the disintegration of the poles and the brilliancy of the metallic spectra. When pure O was passed into the globe and arc the poles were rapidly consumed. The metallic spectrum was very bright, likewise the carbon and cyanogen bands. When the lower hollow carbon was replaced by a carbon with a sulphur core (made by pouring hot sulphur into a hollow carbon) S vapor filled the globe and displaced the air very soon after the arc was started. The metallic lines did not appear, but the carbon and cyanogen bands were strong.

It was thought that the cyanogen bands might have been due to the nitrogen of the air contained in the porous carbon poles. To remove the air the carbons were placed in a small air-tight iron cylinder which was connected by tubes with an air pump and a cylinder of CO_2. The cylinder containing the carbons was placed in a furnace and kept red-hot for two hours; the air pump was worked continuously. The carbons were then allowed to cool in an atmosphere of CO_2, in which they remained for several days. The operation was then repeated and the carbons cooled as before.

When the treated carbons were placed in the lamp and a continuous stream of CO_2 was passed into the arc and globe, the cyanogen bands appeared to be a trifle weaker and the carbon bands a little stronger than with untreated carbon poles. However the difference was very slight. A like result was obtained when, instead of CO_2, O and the vapor of S were passed into the arc. A continuous stream of S vapor was obtained by attaching to the end of the hollow carbon a glass tube sealed at the lower end and filled with S. A Bunsen burner kept the S boiling vigorously, the vapor passing into the globe and arc through the hollow carbon which was heated by a second burner to prevent condensation of the vapor before reaching the arc.

It appears that the weight of evidence favors the view that the three bands in question are intimately connected with the presence of N in the arc. But there are reasons for hesitating to accept the conclusion that they are due to cyanogen or to any other compound of C and N. One reason is that the bands are found in the spectrum of the sun. It is difficult to believe that a carbon nitrogen compound exists at so high a temperature, or even at the temperature of the arc. Another reason is that the strength of the bands does not vary much when the quantity of N in the arc is varied.

All the investigations on this subject show that a mere trace of N is sufficient to bring out the cyanogen bands quite clearly while the carbon bands are weak. Flooding the arc with N does not greatly increase the strength of the cyanogen bands or weaken the carbon bands. The following experiment emphasizes this point:

Mercuric cyanide (30 gms.) was placed in a three bulb combustion tube. One end of the tube was sealed and the other connected by a rubber tube to the hollow carbon of the enclosed arc. The tube was heated by gas burners, and as soon as the cyanogen had displaced all the air in the globe the arc was started. The three cyanogen bands were very slightly, if any, stronger than in the arc in air, and not very much stronger than when treated carbons were used with CO_2 streaming into the arc and globe. Yet the quantity of cyanogen in the arc must have been hundreds of times greater in the first case. That the two spectra should be at all comparable in intensity is not in agreement with our knowledge of spectra in general.

The author inclines to the view that *the so-called cyanogen bands are due to carbon in the presence of nitrogen, but not combined with it.* It is well known that the spectrum of an element is sometimes greatly changed in the presence of another element, though there be no tendency for the elements to combine. The statement need not be confined to spectra. A very familiar instance is the addition of MnO_2 to $KClO_3$ in the generation of O. The temperature at which the O is given off is greatly reduced although the MnO_2 does not decompose nor does it combine with the K.

It has been shown that cyanogen is produced by an arc in air or N. However, the compound may be formed in the outer cooler portions of the arc and not in the central hot region where the cyanogen bands are strongest. The fading out of these bands in the outer regions is more rapid than would be expected if cyanogen were a stable compound in the arc. And to account for their production by a mere trace of N there must be a decided tendency for the compound to form. But experiment shows that cyanogen may be decomposed in the arc.

Copper rods were substituted for the carbons of the enclosed lamp. The lower rod was hollow. Cyanogen was passed through it into the arc and globe. The cyanogen bands were strong. *The carbon bands at* $\lambda=4737$ *and* $\lambda=4382$ *appeared as bright as in the enclosed arc between carbon terminals.*

It may be urged that the decomposition of the cyanogen was brought about by the presence of Cu, and not by heat alone. If this be true it would seem that the metallic impurities ordinarily present in carbon would prevent the formation of cyanogen in an arc where N was present in traces only.

Many substances besides N appear to affect the spectrum of carbon. Sulphur is an instance. When sulphur vapor was forced into the arc it seemed to tend to equalize the band spectrum, somewhat diminishing the intensity of the lines on the side of the bands next the heads, and strengthening the weaker lines on the more refrangible side. The bands were widened until the "grating effect" became continuous in the region of the spectrum photographed. S vapor alone, when forced into an arc between copper poles, gave a faint "grating effect" in the same region.

The effect of one substance upon the spectrum of another has not yet been studied systematically. Liveing and Dewar[1] found that the spectra of mixed vapors differed from the spectra of the substances themselves. Hartley[2] studied the spectra of moistened graphite poles surrounded by various gases, and when immersed in different solutions. He found that the carbon spectrum was modified most by saline solutions, and seemingly in proportion to their strength. Miss Baldwin[3] found that the band spectrum of the arc was weakened by the more positive metals, such as K, Na, Li, Ba, Ca and Sr. Bohn[4] asserts that "it is not possible to ascribe to carbon compounds *any* definite band spectrum." In reviewing Hartley's paper referred to above, Ames[5] remarks that "there is at the present time no more fruitful field open to research than that of the study of the influence of the presence of one substance upon the spectrum of another."

This investigation was made under the direction of Professor Edward L. Nichols, to whom I would express my thanks for the aid extended me and for the facilities placed at my disposal.

PHYSICAL LABORATORY, CORNELL UNIVERSITY, May, 1897.

[1] Proc. Roy. Soc., 34, p. 428.
[2] Phil. Trans., Part I., 1884, p. 49. Proc. Roy. Soc. 55, p. 344, 1894.
[3] PHYSICAL REVIEW, III., p. 448, 1896.
[4] Ztschr. f. physik. Chem., 18, p. 219, 1895.
[5] Astrophysical Journal, I., p. 88, 1895.

CERTAIN PROPERTIES OF HEAT WAVES OF GREAT WAVE-LENGTH. II.

(*Concluded.*)

BY H. RUBENS AND E. F. NICHOLS.

Rocksalt.

THE experiments with rocksalt were by far the most difficult undertaken and have proven, so far, only partially successful. The difficulty in obtaining adequately plane and highly polished surfaces, and their preservation in use, from atmospheric deterioration, was very great. In this we, at last, succeeded by employing a number of unusually pure crystals of salt from Strassfurt, and finally obtained five or six faultless surfaces which withstood, unprotected, during several weeks any considerable deterioration.

Rays from the zirconium lamp after reflection on three such surfaces retained only about one ten-thousandth of their original energy and were still able to traverse thin rocksalt plates with no greater apparent hindrance than ordinary heat rays suffer. Of rays after four reflections, 60 per cent. could still traverse considerable thicknesses of rocksalt. After five reflections rays were finally obtained, only 10 per cent. of which passed unabsorbed through thin rocksalt plates. The five reflections so reduced the energy present that the bolometer gave a throw of only 5 mm. for the total energy. From this result, the necessity of eliminating all diffused and accidental radiation from the bolometer strips, was apparent. To accomplish this, the following arrangement of apparatus was chosen: The rays from the zirconium plate *a* (Fig. 7), were rendered slightly convergent by a concave minor *b* and then reflected from a rocksalt surface at p_1, which reduced the energy to one-twentieth of its original value, after which the rays entered a black wooden box containing the remaining four rocksalt surfaces (p_2—p_5) and the bolometer. Two wooden partitions were so arranged within the box, as to effectually cut off any rays from reaching the exposed bolometer which

had not come from the lamp, and suffered five successive reflections on rocksalt surfaces. Under these conditions, the bolometer did good service and held very steadily to a fixed zero point, so that out of a series of ten observations the mean could be regarded as accurate to 0.2 mm.

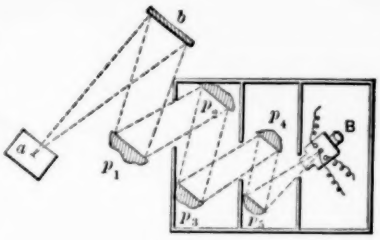

Fig. 7.

With this small quantity of energy it seemed a hopeless task to search through the diffraction spectrum for the maximum necessary to a direct measurement of the wave length. Other experiments described in Part III made it possible, in another way, to reach an estimate of the wave length involved.

Other Substances.

We were not successful in discovering bands of metallic reflection in any of the remaining seven substances, but the search was conducted with the radiometer with a chloride of silver window, so that the finding of any bands lying farther out than the end of the transmission spectrum of silver chloride would have been impossible. From these observations we are therefore only justified in concluding that these substances possessed no regions of metallic reflection for wave lengths less than 25 μ, but it is quite possible that further search by the bolometric method just described might show regions of metallic reflection lying still farther out.

III.

Of the heat rays of various wave lengths already discussed, we have selected those resulting from four reflections on fluorite with which to make a study of the absorption, reflection and refraction of a number of different media. In contrast with the rocksalt rays, the rays from fluorite can be obtained in sufficient intensity to measure these coefficients with an accuracy approaching that attained in similar measurements in better known parts of the spectrum.

Absorption.

A study of the transmission of the fluorite rays in a number of substances has been made, and a part of the results are given in Table VII which requires no explanation.

TABLE VII.

Substance.	Thickness in mm.	Transmission in percentage of incident rays.	Substance.	Thickness in mm.	Transmission in percentage of incident rays.
Fluorite . . .	4.4	0.0%	Quartz . . .	2.25	0.0%
Rock salt . .	1.92	11.0	Mica	0.015	9.9
" . .	5.85	2.1	"	0.032	1.3
" . .	14.0	0.0	Glass	0.10	0.4
Sylvine . . .	3.6	34.0	Paraffin . . .	4.4	0.0
" . . .	6.3	17.8	Ebonite . .	2.0	0.0
" . . .	14.3	4.6	Sulphur . . .	2.0	0.0
Silver Chloride	0.25	77.4	Gold leaf . .	approx. 0.0001	0.0
" "	0.45	52.8			
" "	1.7	43.7	Aluminum-leaf .	approx. 0.001	0.0
Quartz	0.018	15.7			

With the exception of silver chloride, sylvine, and rocksalt none of the substances examined showed measurable transmission in layers approaching 1 mm. in thickness. In very thin layers, quartz, mica, and glass show slight transmission. The plates of fluorite, paraffin, ebonite and sulphur, as well as the two very thin sheets of gold leaf, and aluminum were totally opaque.

In the transmission of layers of varying thicknesses of the same substance, agreement with the law of absorption is very incomplete. This is doubtless because the rays in the first place are not strictly homogeneous, and in the case of silver chloride, none of the plates studied were entirely free from air bubbles and enclosed particles of foreign substances so that here the discrepancy between the law of absorption and the observed transmission is especially marked. The behavior of lamp black, carbon dioxide, and water vapor toward these rays has been earlier noted.[1]

Experiments with rays after five reflections on rocksalt surfaces,

[1] Rubens and Nichols, l. c.

show that nearly all substances absorb these rays in even greater measure than those from fluorite. Rocksalt, sylvine, silver chloride, and fluorite in very thin layers absorbed these rays totally, also sulphur, ebonite, glass and quartz, when one or more millimeters in thickness. Of all the substances examined, paraffin was most diathermous, transmitting 50 per cent. through plates 1.5 mm. thick. We hope later by the aid of a radiometer with a thin paraffin window to measure the wave length of these rays.

Reflection.

For the reflection measurements, the arrangement of apparatus shown in Fig. 8 was used, and the measurement was a comparison of the reflection of a plane surface of the substance studied, with that of a polished silver plane mirror.

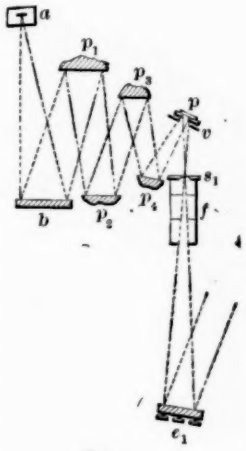

Fig. .

The rays from the burner at *a*, after reflection on the fluorite surfaces p_1-p_4, were reflected from the surface *r* of the body in question into the slit s_1 of the spectrometer. The accurate replacing of the surface *r* with that of the comparison mirror, was accomplished by means of a plane brass plate with two equal apertures, one above the other. This plate slid on a carefully constructed bearing, so that the plane remained unchanged. Two stops, placed on the bearing, allowed the plate to slide over a distance equal to that between the centers of the two circular apertures, so that when the plate struck against the upper stop, the lower aperture was in the same position as the upper aperture when the plate was in contact with the lower stop. Surfaces to be compared were brought in contact with the back surfaces of the plane brass plate behind the apertures. Thus the reflection of each substance studied was compared with silver, and, from time to time, two equal silver mirrors were compared to make sure that the adjustment favored neither of the apertures above the other. To further exclude any inaccuracy which might arise from a slight displacement of the stops, a permanent diaphragm *v* with an opening

smaller than the apertures in the plate, was mounted directly in the path of the rays in front of the movable plate.

The reflection of silver was regarded throughout these measurements as total, an assumption in the justification of which the following considerations may be offered: First, the reflection of silver in the infra red as well as in the visible spectrum has been shown to increase with the wave length. At 3 μ its reflection already exceeds 98 per cent. and at 4 μ is more than 99.[1] Second, the reflection of silver has been carefully compared with that of a number of other metals for rays of 23.7 μ wave length and it has been found that within the limits of observational error the reflection coefficient of every metal studied was equal to that of silver itself. Table I shows the results of these measurements, to which the reflection percentages of a number of the same metals for green light has been added to afford an interesting comparison.[2]

TABLE VIII.

Metal.	Reflection percentage. $\lambda = 23.7\mu$	Reflection percentage. $\lambda = 0.50\mu$
Silver	100.0	88.3
Gold	99.8	56.1
Platinum	100.0	—
Copper	100.7	54.8
Iron	99.5	57.7
Nickel	100.3	61.0
Brass	100.0	—
Speculum metal	100.1	—

From the table it appears probable that, not only the reflection of silver but also that of the remaining metals and alloys have, at this wavelength, practically reached 100 per cent—the theoretical limit.

In the same way the reflection of seven of the substances contained in Table VII was measured. All of these substances, except fluorite, showed, by a slight transmission, that the relation between reflection and refraction in the Fresnel formula held without sensible

[1] H. Rubens, Wied. Ann., 37 S., 249, 1889, and E. F. Nichols, PHYSICAL REVIEW, 297, IV., p. 22, 1897.

[2] H. Rubens, Wied. Ann., 37 S., 265, 1889.

error. As the results could, at best, be only close approximations, the simplified form of the equation—for normal incidence—was used in the computation of n, although the reflection measured took place under an angle of about 13°.

The results appear in Table IX.

TABLE IX.

Substance.	Percentage of reflection.	Index of Refraction n computed.
Fluorite	71.5	—
Quartz	41.7	4.62
Mica	28.7	3.32
Glass	19.7	2.55
Sulphur	11.4	1.95
Sylvine	2.0	1.32
Rock-salt	1.7	1.22

According to the observations, fluorite shows a mean reflection of 71.5 per cent. For rays in the middle of the absorption band the reflection is doubtless much greater, probably rivalling that of silver in the visible spectrum. The high reflection of quartz and mica, and the computed values of the refractive index for these materials, are doubtless due to bands of metallic absorption, which, in both substances, occur in the immediate neighborhood—in quartz at 20.7 μ and in mica at 21.25 μ. In both, the bands lie on the side of the shorter wave lengths. The indices of refraction for glass and sulphur lie very near the square roots of their dielectric constants, the agreement in the former case, however, may well be accidental.

As might well have been expected from earlier computations of their dispersion constants, surfaces of rocksalt and sylvine, by their reflection, show a much lower index of refraction than the square root of the dielectric constants would give. The reason for the discrepancy will be shown later.

Refraction.

The transmission of only three of the substances so far examined—sylvine, rocksalt, and chloride of silver—was sufficient to make the

direct determination of refraction index, by means of a prism, possible. Of these three, silver chloride was excluded on other grounds.

In sylvine and rocksalt the absorption was so great that it was necessary to use prisms of very acute refracting angles—for rocksalt $\varphi = 10^\circ\ 53'$ for sylvine $\varphi = 12^\circ\ 39'\ 10''$. The surfaces of the prisms obtained were plain, and sufficiently polished to permit the determination of the prism angle, accurate to within $10''$, by the Gauss eyepiece. The prism angle and the refractive index for sodium light were determined, in both prisms by the aid of a spectrometer of the usual type. The refractive indices observed agree with the similar measurements, by one of the present writers,[1] to within one unit in the fourth decimal place for rocksalt, and to within two units for sylvine. The agreement is very satisfactory, when the small refracting angle of the prisms is taken into account. The prisms were next placed on the table of the reflecting spectrometer $s_1\ e_1\ e_2\ s_2$, in place of the grating at g (Fig. 1), and by the radiometer, the deviation of the energy maxima for quartz and fluorite rays was observed, first on the right, the prisms reversed, and the same operation repeated on the left. In the quartz spectrum the deviation of the maximum at 20.75μ, and of the maximum in the fluorite spectrum are given for rocksalt in Table Xa, and for sylvine in Table Xb.

TABLE Xa.

ROCK-SALT.

Refracting angle = 10° 53′ 0″

Series.	Band I (Quartz)	Band II (Fluorite)
I	$R = 2^\circ\ 27\frac{1}{2}'$	$R = 2^\circ\ 14\frac{1}{2}'$
	$L = 2\ \ 28$	$L = 2\ \ 15\frac{1}{2}$
II	$R = 2\ \ 28$	$R = 2\ \ 15$
	$L = 2\ \ 29$	$L = 2\ \ 15$
Mean	2° 28′ 8″	2° 15′ 0″

[1] H. Rubens, Wied. Ann. 45, p. 238, 1892, and H. Rubens and B. W. Snow, Wied Ann. 46, p. 529, 1892.

TABLE Xb.

SYLVINE.

Refracting angle = 12° 39′ 10″

Series.	Band I (Quartz)	Band II (Fluorite)
I	$R = 2°\ 59'$ $L = 3\ \ 0$	$R = 2°\ 51'$ $L = 2\ \ 51$
II	$R = 2\ \ 59$ $L = 2\ \ 59\frac{1}{2}$	$R = 2\ \ 50$ $L = 2\ \ 50\frac{1}{2}$
Mean	2° 59′ 22″	2° 50′ 37″

The letters R and L in the tables denote deviations to the right and to the left sides respectively. The deviation of the sodium line picked up by the radiometer was 3° 35′ 58″ for the rocksalt prism and 3° 46′ 40″ for the sylvine prism. These deviations were all measured with the spectrometer arranged with the fixed slit s_2 (Fig. 1), and as has already been described in Part I, must be multiplied by a constant factor to gain the corresponding true deviations δ. This factor was redetermined by the aid of the grating, after the present observations were made and was found to be 1.6540.[1] The values of δ obtained by multiplication from values of α, together with the refractive indices corresponding to the computed deviations, are given in Tables XIa and b.

TABLE XIa.

ROCK-SALT.

	Observed deviation α	Corrected deviation δ	n
Sodium	3° 35′ 58″	5° 57′ 15″	1.5437 (1.5439)
Band I. (Quartz) . .	2 28 8	4 5 0	1.3735
Band II. (Fluorite) .	2 15 0	3 43 18	1.3403

[1] The difference between this value and the value 1.6522 on page — is due to a slight change in the position of the slit s_2.

TABLE XIb.

SYLVINE.

	Observed deviation α	Corrected deviation δ	n
Sodium	3° 46′ 40″	6° 15′ 0″	1.4902 (1.4905)
Band I. (Quartz) . .	2 59 23	4 56 42	1.3882
Band II. (Fluorite) .	2 50 37	4 42 12	1.3692

The values of n for the sodium line which appear in parenthesis were the values observed with the ordinary spectrometer mentioned earlier. This agreement of the values obtained by the two methods serves as a check upon the accuracy of the measurements here given.

A more exact determination of the wave lengths of the maxima of the two bands I. and II. in the prismatic spectrum requires not only a knowledge of the dispersion in those regions, but also a knowledge of the selective absorption in both cases. It is, however, only necessary to know these approximately, in order to gain a very close estimate of the wave lengths involved, especially as concerns band I., where the correction to the wave length, as measured in the grating spectrum, will be less than 1 per cent. The displacement of grating spectrum maximum of band I. due to the absorption of the prisms as measured by one of the present writers, was for rocksalt 0.10μ and for sylvine only 0.05μ—a wave length correction of 0.5 and 0.3 per cent. respectively. The effect of selective absorption to displace the maximum of band II., was much greater in both cases, for rocksalt 1.1μ and for sylvine 0.9μ.

The correction for varying dispersion in these regions is small—for band I. 0.4 per cent., for band II. 1.2 per cent. The corrections were made by plotting the dispersion curves for both prisms. This was done by using the wave lengths of both bands, corrected for absorption, and the corresponding indices of refraction, and also, by the use of the earlier measurements of dispersion by one of the present writers. The dispersion curve gives values sufficiently exact of the differential coefficient $dn/d\lambda$ by which the ordinates of

the energy curve of the grating spectrum were divided, and the distribution of energy in the prismatic spectrum obtained. This presupposes that $dn/d\lambda$ is proportional to $d\delta/d\lambda$ which, taken strictly, is true, only for a prism of infinitesimal refracting angle, in which case

$$n=\frac{\sin\frac{\varphi+\delta}{2}}{\sin\frac{\varphi}{2}}=\frac{\varphi+\delta}{\varphi}$$

$$\text{and } \frac{d\delta}{d\lambda}=\frac{d\delta}{dn}\frac{dn}{d\lambda}=\varphi\frac{dn}{d\lambda}$$

These equations still hold with a sufficient approximation for $\varphi=12°$, to justify their use in the present case. When all the corrections are taken into account, the wave lengths of the maxima of bands I. and II. in the rocksalt prism were at $\lambda=20.57\mu$ and 22.3μ, and for the sylvine prism at $\lambda=20.60\mu$ and 22.5μ respectively. The accuracy of these values is very unequal. The probable error for band I. is, in both prisms, about 0.3 per cent., while for band II. the results can scarcely be regarded as certain within 1.0 per cent.

In the computation of the constants in the Ketteler-Helmholtz dispersion formula (1) which we sought to determine in connection with these values, only the values for band I. were used in addition to the earlier dispersion measurements by one of the present writers. The constants M_1 and λ_1 which have very little influence upon the course of the curve beyond $7.\mu$, were taken from the earlier measurements. The values of M_2 and λ_2, on the other hand, were newly determined from two values of the refractive index in the infra-red spectrum, one of which was the new value for band I. The constant b^2 was finally determined by addition from a^2 and M_2/λ_2^2. In Tables XIIa and b, the observed indices of refraction for a number of wavel engths in the visible and infra-red spectrum, are compared with corresponding values computed by the aid of equation (1). As an interesting comparison the values computed from the simplified formula:

$$n^2=a^2+\frac{M_1}{\lambda^2-\lambda_1^2}-k\lambda^2 \qquad (3)$$

and the constants of the earlier measurements are given also. The numbers turn out much too large for the greater wave lengths, while values from (1) agree satisfactorily with all observed values.

TABLE XIIa.

ROCK-SALT.

λ	n observed.	n comp. (1)	n comp. (3)
0.434μ	1.5607	1.5606	1.5606
0.589	1.5441	1.5441	1.5441
8.67	1.5030	1.5030	1.5030
20.57	1.3735	1.3735	1.3926
22.3	1.340	1.3403	1.3679

$b^2 = 5.1790$, $M_1 = 0.018496$ $M_2 = 8977.0$
$\lambda_1^2 = 0.01621$, $\lambda_2^2 = 3149.3$

TABLE XIIb.

SYLVINE.

λ	n observed.	n comp. (1)	n comp. (3)
0.434μ	1.5048	1.5048	1.5048
0.589	1.4900	1.4899	1.4899
7.08	1.4653	1.4653	1.4654
20.60	1.3882	1.3882	1.3948
22.5	1.369	1.3688	1.3789

$b^2 = 4.5531$, $M_1 = 0.0150$, $M_2 = 10747$
$\lambda_1^2 = 0.0234$, $\lambda_2^2 = 4517.1$

Of the constants determined for rocksalt and sylvine from equation (1) λ_2 is important, for it must represent approximately the wave length of the infra-red absorption band in these materials and consequently the wave length of rays after multiple reflection on these substances. Hence the wave length of rays, after five reflections from rocksalt, should be $\sqrt{3149.3} = 56.\mu$. From the relation between the values of λ_2 from equation (1) for quartz and fluorite, to

the observed values, it is probable that 56.μ is too great, and that the wave length, when measured, will be found not far from 50.μ.

As has been pointed out by F. Paschen, the constant b^2 should represent the dielectric constant for each substance, as it is the square of the index of refraction for an infinite wave length. We are now in position to add three more substances to the list containing the value of b^2 observed by Paschen for fluorite, and for flint glass by one of the present writers.

These values are given in Table XIII, together with values of the dielectric constant as observed by other methods. The names of the observers are added in the last column.

TABLE XIII.

Substance.	b^2	Dielectric constant κ	Observer.
Flint-glass	6.77	6.7 — 9.1	Hopkinson.[1]
Fluorite	6.09	6.8	Curie.[2]
		6.7	Romich u. Nowak.[3]
		6.9	Starke.[4]
Quartz	4.58	4.55	Curie.
		4.6	Romich u. Nowak.
		4.73	Starke.
Rock-salt	5.18	5.85	Curie.
		5.81	Thwing.[5]
		6.29	Starke.
Sylvine	4.55	4.94	Starke.

The closest agreement between b^2 and the dielectric constant occurs in quartz in which the data for the computation of the dispersion constants are fullest. When the uncertainty connected with the computation of b^2 and also in the methods for finding the dielectric constant by direct observation taken into account the agreement in the remaining instances is as good as could have been expected.

[1] Hopkinson, Phil. Trans. 2, p. 355, 1881.
[2] Curie, Ann. Chim. Phys. (6) 17, p. 385, und 18, p. 203, 1889.
[3] Romich u. Nowak, Wien. Ber. (2) 70, p. 380, 1874.
[4] H. Starke, Dissertation, Berlin, 1896.
[5] H. Rubens, Wied. Ann. 45, p. 238, 1892.

IV.

Electro-magnetic Experiments.

Encouraged by the relatively great length of the rays from fluorite the success of an investigation to discover electrical resonance seemed by no means impossible. The experiments made were closely analogous to those performed by A. Garbasso[1] in 1893, in which the reflection of electrical waves from wooden walls covered with a number of uniformly distributed parallel electrical resonators was studied. Garbasso found that such walls showed a high reflection percentage only when the period of the vibrator was equal to that of the resonators. The study was made with electrical waves 43 and 70 centimeters long.

The production of plates of resonators bearing the same relation to the lengths of our waves which, although relatively very long, were in reality very short was attended by the greatest difficulties which only after many unsuccessful attempts we were able in a measure to overcome.

A number of glass plates with plane surfaces were silvered by the usual chemical process. Each immediately after silvering was ruled with a diamond point on the dividing engine into a grating with 100 lines to the milimeter. The width of the line cut through the silver coating was made as nearly as possible equal to the width of the remaining strip of silver so that both were approximately 5μ wide. It was usually possible to attain this end with sufficient exactness by carefully choosing the diamond point and by regulating the pressure for each individual silver coating supposed in itself to be uniformly coherent.

Gratings obtained in this way were turned through a right angle and ruled with lines perpendicular to the former ones. The grating was thus cut into a great number of minute rectangular fields of silver to serve as resonators. No cost in patience or time was spared to make these fields for each plate as nearly uniform as possible. Five such plates were finally obtained with resonators of as many different lengths. The length and width of the individual resonators and the number to each square centimeter of surface are given in Table XIV.

[1] A. Garbasso, Accad. delle Scienze di Torino, 1893. Vol. XXVII.

Of the five plates of resonators which finally came to measurement, one was a grating, *i. e.*, contained resonators which compared with the wave length were practically of infinite length. The remaining four plates were covered with resonators the length of which varied from plate to plate by about 6μ. The surface on each plate covered with resonators was $(1.\ 5\ \text{cm.})^2$ or $2.25\ (\text{cm.})^2$.

The study of a greater number of plates of resonators which would have been in many respects desirable was hindered by the great difficulties encountered in ruling the plates, chief of which was the difficulty found in getting silver coatings of uniform coherence on different portions of the same plate. By far the greater part of the resonator plates made according to the method described were found on examination to be useless, either the diamond point had not cut through the silver coating at all points, or the metal in many places had been torn entirely away. Both faults often occurred on the same plate. The resonators on the plates finally studied were by no means everywhere intact yet with the exception of Plate V., the proportion of injured resonators was not large enough to materially affect the results. Of the resonators on Plate V. nearly 10 per cent were imperfect so that the reflection obtained for this plate was unquestionably too small. The cross rulings in this grating were far enough apart ($\lambda+5\mu$,—the length of the resonator plus the width of the line) so that diffraction images of the first order were possible; a source of error from which the other plates were entirely free. The effect of this error upon the measured reflection of Plate V. would also be to make it too low by the amount lost through diffraction.

The same arrangement of apparatus shown in Fig. 8 was used here except that a plane glass plate was so introduced into the path of the rays between p_4 and r that the middle rays in the bundle were reflected at the angle of complete polarization[1] before reaching the resonators at r.

In every case the reflection of the resonator plates was studied in two positions at right angles to each other. First with the electrical

[1] The index of refraction n of the glass plate d was computed from its reflection percentage ρ by means of the Fresnel formula. In this case $\rho=20.2$ per cent. and $n=2.62$. The transmission of very thin plates of glass for these rays was sufficient to warrant neglecting the absorption in the computation of n.

component of the polarized rays parallel to the long way of the resonators, and second, with the electrical component and long way of the resonators at right angles.

Table XIV gives the results obtained and also the dimensions and characteristics of the resonators of the different groups.

TABLE XIV.

Number of Resonator Plate.	I	II	III	IV	V
Number of Resonators per sq. centimeter	1×10^3	1000×10^3	572×10^3	400×10^3	333×10^3
Length (l) of Resonators	∞	6.5μ	12.4μ	18.0μ	24.4μ
Width (w) of Resonators	5.8μ	4.6μ	5.3μ	5.1μ	5.5μ
Reflection percentage electrical component perpendicular to long way	40.8	38.3	42.7	40.7	36.1
Reflection percentage electrical component parallel to long way of Resonators	83.7	41.8	65.8	49.5	62.5

The values obtained for the reflection of the different plates may be regarded as consisting of two parts, the one due to the reflection of the resonators themselves and the other due to the reflection of the glass plate. The latter component is by no means negligible, for in the present case it may reach 24.7 per cent., which was shown by repeated measurements to be the reflection of the glass plates before silvering.

To get at the approximate reflection of the resonators independent of this background we may proceed as follows: Let α be the observed reflection percentage of resonators and glass combined. Let β be the part of this due to the resonators alone and γ the part reflected by the glass, then

$$\alpha=\beta+\gamma.$$

The part not reflected by the resonators $100-\beta$ will be the intensity

[1] The index of refraction n of the glass plate was computed from its reflection percentage ρ by means of the Fresnel formula. In this case $\rho=20.2$ per cent. and $n=2.62$. The transmission of very thin plates of glass for these rays was sufficient to warrant neglecting the absorption in the computation of n.

of the rays incident on the glass, and of this amount 24.7 per cent. will be reflected; hence

$$\gamma=(100-\beta)\,0.247$$

and

$$\beta=\frac{\alpha-24.7}{0.753}$$

The formula rests upon two assumptions which have never been proven: First, that the resonators do not absorb and, second, that the glass surface and resonators reflect independently, neither disturbed by nor disturbing the other. The formula is in consequence unassailable only in the limiting cases where $\beta=0$ or 100, and between these limits it may be regarded as an interpolation formula, giving only a first approximation to true values.

From the values of α in Table XIV values of β have been computed by the formula and are given in Table XV.

TABLE XV.

Number of resonator plate.	I	II	III	IV	V
β, Electrical component perpendicular to long way of resonators	21.4	18.1	23.9	21.3	15.3
β, Electrical component parallel to long way of resonators	78.4	22.7	54.5	32.9	50.2

A glance at the table shows that the values in the line representing the reflection of the different resonator plates varied between comparatively narrow limits when the electrical component was perpendicular to the long way of the resonator. If for reasons already given the value for Plate V be excluded, the limits of variation become 18.1 and 23.9 per cent. On the other hand, the numbers in the line showing the reflection when the electric component was parallel to the long way of the resonators show great variation, not only when compared with corresponding values in the line above, but also among themselves. The result of the comparison leads to the following conclusions:

1st. All the resonators show a higher reflection when the electric vector and the long way of the resonators coincide, than when at right angles to each other.

2d. The resonators of Plates I, III and V show a much stronger reflection than Plates II and IV when the electric vector lies in the long way of the resonators; when at right angles the difference is small.

3d. The highest reflecting power is exhibited by the resonators of Plate I which, in comparison with the incident wave length, may be regarded as infinitely long.[1]

The results may easily be brought into satisfactory agreement with the requirements of the electro-magnetic theory of light. We can think of the long resonators of Plate I. as divisible with sufficient accuracy by a very great whole number of half wave lengths for all the observed wave frequencies between $\lambda=22$ and 28μ and thus these resonators are theoretically to be regarded as representing the most favorable conditions for perfect resonance.

The remaining results are further in agreement with results reached by earlier experiments[2] with electrical waves in which a higher degree of resonance was observed when the resonators approached a whole number of half wavel engths than when near an odd number of quarter wave lengths. The length of the resonators of Plates II, III, IV and V represented approximately 1, 2, 3 and 4 quarter wave lengths respectively.

That the observed resonance phenomena were not even more sharply defined and definite than they appear here can doubtless be accounted for by a more careful consideration of the conditions under which the experiments were made. In the first place we have no assurance that the period of the resonators of plates III and IV corresponded to that of the energy maximum of the incident rays. The mathematical computation of the length of maximum resonance for resonators of the form used has never been made and it is questionable if in such a computation one may neglect the influence of each resonator upon the period of its neighbors and vice versâ. Second, the relation of length to width in the resonators used would probably lead to a heavily damped vibra-

[1] That gratings of parallel wires exert a similar polarizing action upon transmitted infra-red rays was shown in 1892 by Dr. Du Bois and one of the present writers—H. Rubens, (Wied. Ann., 49, S593, 1893). The polarizing action then observed, because of the shorter waves employed, was very much smaller.

[2] A. Righi, Rendic. R. Acc dei Lincei (6) 2, p. 505, 1893.

tion due to a relatively large capacity and low self-induction. Third, the incident beam was a convergent bundle and therefore only the middle part could have been completely polarized by reflection. Lastly, the incident rays were not strictly homogeneous.

Under more favorable circumstances we think a higher degree of electrical resonance might confidently be expected, although we believe that electrical resonance as a property of heat rays has been proved beyond question by the experiments here described.

The writers, in closing, wish to express their appreciation of the kindness of President F. Kohlrausch and of Professors E. Warberg and A. Kōnig who lent several pieces of valuable apparatus used in carrying out the present study.

We wish further to thank Mr. Augustus Trowbridge for kindly assistance in the ruling of a part of the resonator plates.

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THE SURFACE TENSION OF WATER AND OF CERTAIN DILUTE AQUEOUS SOLUTIONS, DETERMINED BY THE METHOD OF RIPPLES. I.

BY N. ERNEST DORSEY.

INTRODUCTION.

PREVIOUS observers have found that the surface tension of most aqueous solutions is a linear function of the concentration; but they have worked on no solution more dilute than about one-half normal (a normal solution is one that contains in one liter of the solution as many grams of the salt as there are units in the molecular weight of the salt, the molecular weight of hydrogen being taken as two). It seems, therefore, desirable to determine, if possible, the surface tension of more dilute solutions; and this work was undertaken at the suggestion of Professor J. S. Ames in the spring of 1895, for the purpose of studying solutions as dilute as possible. Most of the time since then has been spent in studying the various methods, and in perfecting the method of ripples, which appeared to offer fewer theoretical objections than any other.

The arrangement of apparatus finally adopted gives an average departure from the mean of several observations of not more than one-seventh of one per cent. Though different samples of water may give results that differ among themselves by more than this amount, still in the entire series of twenty-one samples of water the average departure of a single observation from the mean was but one-fifth of one per cent. This compares very favorably with the results obtained by any other method.

This spring determinations of the surface tensions of a few solutions were made at concentrations varying from one-tenth normal to normal, and the values found lie upon straight lines starting from the value for pure water.

Historical and Critical.

Among those who have worked on the surface tension of aqueous salt solutions may be mentioned Frankenheim,[1] Buliginsky,[2] Valson,[3] Quincke,[4] Lippmann,[5] P. Volkmann,[6] O. Rother,[7] Traube,[8] Röntgen and Schneider,[9] Klupathy,[10] Goldstein,[11] Jäger,[12] N. Kasanskine,[13] Canestrini,[14] Sentis.[15]

Excepting Sentis, Jäger, and Klupathy, they all used the method of capillary tubes, and Quincke supplemented this with his method of large bubbles. All determinations of the surface tension from the measured rise of the liquid in capillary tubes involve an assumption in regard to the contact angle, *i. e.*, in regard to the angle included between the wall of the tube and the tangent to the liquid surface at the *point of contact with the tube.* This angle cannot be measured, for by any possible method we must measure the inclination of a finite arc, and as the surface extends on one side only of the point at which we desire the inclination, the finite arc whose inclination we measure must lie entirely on one side of the point of contact. Hence every measured value of this angle will necessarily be too large. This is probably the reason Professor Quincke obtains finite contact angles for liquids that wet the glass in contact with them.

Another objection to measuring the surface tensions of solutions at the line of intersection of three bodies, solution, air and glass, is that probably the tensions of both the solution-air and the solution-glass surfaces change with the concentration of the solution, so that the results obtained involve two changes which can not be separated. This renders the interpretation of the results very diffi-

[1] Pogg. Ann., 37, 409, 1836.
[2] Pogg. Ann., 134, 440, 1868; A. ch. ph. (4), 20, 361, 1870.
[3] C. rend., 74, 103, 1872.
[4] Pogg. Ann., 160, 337 and 560, 1877.
[5] Wied. Ann., 11, 316, 1880.
[6] Wied. Ann., 17, 353, 1882.
[7] Wied. Ann., 21, 576, 1884.
[8] Jour. f. pr. ch., 31, 192, 1885.
[9] Wied. Ann., 29, 165, 1886.
[10] Math. u. naturwiss Ber. aus Ungarn, 5, 101, 1887; Wied. Ann. Beibl., 12, 750, 1888.
[11] Zeitschr. f. phys. chem., 5, 233, 1890.
[12] Acad. d. Wiss. in Wein, 100_{2A}, 493, 1891.
[13] J. de la Soc phys. chem. russe, 23, 468, 1891; Jour. de Phys. (3), 1, 406, 1892.
[14] Riv. Sc. ind., p. 33, 1892; Wied. Ann. Beibl., 16, 335, 1892.
[15] Journ. de Phys. (2), 6, 571, 1887; Thesis published at Paris, February, 1897 Jour de Phys. (3), 6, 183, 1897.

cult. Jäger used a method depending on the relation between the diameter of a capillary tube and the distance it must be immersed in the liquid in order that the same air pressure might cause steady streams of bubbles of the same size to issue from it and from another tube of known diameter immersed to a fixed depth. He determined this relation empirically from experiments on water, and he accepted the results of Brunner and of Wolf as correct. Quincke's work on flat bubbles has been discussed in several papers and seems to be not very satisfactory. From this we see that, excepting the work done by Sentis and by Klupathy, the determinations of the surface-tensions of solutions are not all that can be desired.

Sentis employed an entirely different method, and one that appears to be very free from objections. He draws the liquid up into a capillary tube, then removes the tube from the liquid and allows the liquid to run out so as to form a drop around the end of the tube. This drop supports a certain length liquid column in the tube. He then measures the maximum diameter of the drop, focuses a microscope on the meniscus in the tube, and slowly raises below the tube a beaker of the liquid. When the liquid in the beaker reaches the drop, the column in the tube falls; the position of the beaker is now noted and it is then again slowly raised until the meniscus in the tube has come back to its previous position, and the position of the beaker is then noted. The vertical distance between these two positions of the beaker is (after certain corrections are made) the height the liquid will rise in a tube for which the contact angle is zero and whose radius is equal to that of the drop. His individual determinations may differ by one-half of a per cent., but generally they agree very well.

In the present work it was desired to use a method entirely independent of the mutual action between the liquid and a solid; and the only methods so far devised that completely satisfy this condition are Lenard's[1] method of vibrating falling drops, the method of Eötvös,[2] that employed by Sentis, and the method of ripples, which was first successfully used by Lord Rayleigh.[3] Sentis' method, as

[1] Wied. Ann., 30, 209, 1887. [2] Wied. Ann., 27, 448, 1886.
[3] Phil. Mag. (5), 30, 386, 1890; Theory of Sound, Vol. II., p. 344.

described in his first paper[1] did not seem very satisfactory; and, of the others mentioned, Lord Rayleigh's appeared to be the most promising, and is the one finally adopted.

Method of Ripples.

This method is based on the relation between the surface tension of a liquid and the velocity of propagation, on its surface, of a train of small waves of given period. This was deduced by Lord Kelvin[2] for the case of infinitely small waves on the surface of a perfect liquid of given depth. His formula is

$$V^2=\left(\frac{g\lambda}{2\pi}+\frac{2\pi T}{\rho\lambda}\right)\tan\mathrm{h}\,\frac{2\pi h}{\lambda}.$$

Here V is the velocity of propagation; λ is the wave-length; T is the surface tension; ρ is the density; h is the depth of the liquid; and g is the acceleration of gravity. If n is the frequency of the waves we can write this equation thus:—

$$T=\rho\left(\frac{\lambda^3n^2}{2\pi}\cot\mathrm{h}\,\frac{2\pi h}{\lambda}-\frac{g\lambda^2}{4\pi^2}\right).$$

Now $$\cot\mathrm{h}\,\frac{2\pi h}{\lambda}=\frac{1+\varepsilon^{-\frac{4\pi h}{\lambda}}}{1-\varepsilon^{-\frac{4\pi h}{\lambda}}}=1+2\varepsilon^{-\frac{4\pi h}{\lambda}}$$

approximately if h is large.

In this work h is never less than 1.1 cm., and λ is never as great as 0.5 cm. Hence

$$\frac{4\pi h}{\lambda}\not<25\quad\therefore\cot\mathrm{h}\,\frac{2\pi h}{\lambda}\not>1+2\varepsilon^{-25}.$$

This shows that we may consider $\cot\mathrm{h}\,\frac{2\pi h}{\lambda}=1$, and may write the equation

$$T=\rho\left(\frac{\lambda^3n^2}{2\pi}-\frac{g\lambda^2}{4\pi^2}\right).$$

[1] Journ. de Phys. (2), 6, 571, 1887.
[2] Phil. Mag. (4), 42, 375, 1871.

Lamb[1] gives the more exact formula for the case of a liquid of infinite depth

$$V=\left(\frac{\rho-\rho'}{\rho+\rho'}\,\frac{g\lambda}{2\pi}+\frac{2\pi T}{(\rho+\rho')\lambda}\right)$$

or

$$T=(\rho+\rho')\left(\frac{\lambda^3 n^2}{2\pi}-\frac{\rho-\rho'}{\rho+\rho'}\,\frac{g\lambda^2}{4\pi^2}\right)$$

where ρ' is the density of the air in contact with the surface. For the accuracy obtained in measuring λ the first and simpler formula is sufficiently exact.

Of the terms on the right of these equations, the first is much the larger for small waves. Hence the value calculated for T will be proportional to the cube of the wave-length and to the square of the frequency. Here is the first objection to this method. The value found for T will be only one-third as accurate as the measurement of the wave-length, and one-half as accurate as the determination of the frequency. Again, since the formulæ are deduced for waves of infinitely small amplitude, we must use very moderate vibrations and these are difficult to measure.

Several, among whom we need mention only Matthiessen, Riess, and Ahrendt, have attempted to test experimentally the extreme accuracy of Lord Kelvin's formula.

When a needle is placed so that its point dips slightly into a jet of water, a series of stationary waves is formed behind the needle, and their length is equal to that of waves whose velocity is equal to the velocity of the surface of the jet at that point. This is the method employed by Ahrendt.[2] He found that the value of the surface tension calculated from the observed wave-length is too great but that it decreases as the needle is removed from the orifice. This proved that the fault lay in assuming that the surface of the jet moves with the velocity calculated for the ideal case. In reality, the surface has a smaller velocity, probably owing to friction with the edge of the orifice and with the air. Hence he proved nothing, so far as the formula is concerned.

Both Reiss[3] and Matthiessen[4] failed to satisfy two of the conditions

[1] Hydrodynamics, p. 446.

[2] Exner's Rep. der Phys., 24, 318, 1888.

[3] Exner's Rep. d. Phys., 26, 102, 1890.

[4] Pogg. Ann., 134, 107; 141, 375; Wied. Ann., 32, 626; 38, 118.

which were assumed by Lord Kelvin in obtaining his formula. One of these is that the amplitude of the waves shall be very small in comparison with the wave-length; and the other is that the surface of the liquid may be represented by an equation of the form

$$y = a \sin (mx + nt)$$

i. e., the formula is deduced for the case of a single series of very small, plane waves. The waves employed by these observers were of such an amplitude that they could be easily seen with the unaided eye; this alone might very probably introduce a discrepancy between the observed and calculated values for the wave-length. This applies to all measurements made by them. Most of their observations were made on the interference waves produced on the surface of a liquid when a vibrating fork is supported vertically above it so that two needles, one attached to either prong, may dip into the liquid. This gives two series of circular waves and so the second condition mentioned above is not fulfilled. I do not know how much this would affect the result, but until it is proved to have no affect the results must be considered as nugatory. Riess[1] says "Der Schlüssel für die definitive Lösung ist in einer minutiösen Messung der Amplituden zu suchen" and he goes on to say that the only sure method is an optical or photographic one. Matthiessen found that the formula gives results in accord with his observations except in the neighborhood of the minimum value of the velocity.

C. M. Smith[2] employed this method for determining the surface tension of mercury, but obtained very erratic results. Lord Rayleigh[3] thinks he obtained waves due the suboctave of his fork, and these are almost sure to have too large an amplitude.

W. Ochse[4] attempted to determine the surface tension of solution by means of ripples. He measured the interference pattern of two series of circular waves, as Riess and Matthiessen had done, his waves had a large amplitude, he assumed that the density did not

[1] Exner's Rep. d. Phys., 26, 131, 1890.
[2] Proc. Roy. Soc. Edinb., 17, 115.
[3] Phil. Mag. (5), 30, 386, 1890.
[4] Exner's Rep. d. Phys., 26, 641, 1890.

enter into the equation, and he used rather concentrated solutions. His measurements of the wave-length are quite rough, so that the results have but little value even when corrected for the density.

From this it is seen that the formula has not been disproved, and the fact that Lord Rayleigh obtained the same value of the surface tension of water with two forks of different frequencies, and that the result obtained for water in this work agrees, within his experimental error, with Lord Rayleigh's seems to justify the application of Lord Kelvin's formula to these two cases.

All the work so far described has been with waves easily visible, and these very probably have an amplitude too large to allow the application of Lord Kelvin's formula. Lord Rayleigh was the first to use waves invisible under ordinary conditions. This method was to have the waves generated by means of a plate of glass which was attached to the lower prong of a vibrating fork, and which dipped into the liquid to be experimented upon. Then "in order to see the waves well, the light was made intermittent in a period equal to that of the waves, and Foucault's optical method, for rendering visible small departures from truth in plane or spherical reflecting surfaces, was employed." The liquid was contained in a shallow porcelain tray, and its surface was cleaned by a flexible brass hoop wider than the depth of the liquid. This was placed in the liquid so as to include the plate that generates the ripples, but otherwise in as contracted a position as possible. It was then expanded to its maximum extent. This gives a fresh surface as clean as the body of the liquid. The mean of his results for water is $T=74$ dynes per centimeter at 18° centigrade, and this is probably correct to about one per cent.

DESCRIPTION OF APPARATUS.

I shall now describe the apparatus used in this work, and which is simply a natural development of Lord Rayleigh's, although it contains many new features.

The arrangement of apparatus is represented diagrammatically in Fig. 1. S is a sixteen candle powder incandescent electric lamp; S_2 consists of two narrow strips of thin copper, one of which is fastened to each prong of the large tuning fork (F_1) (see Fig. 2), and each has

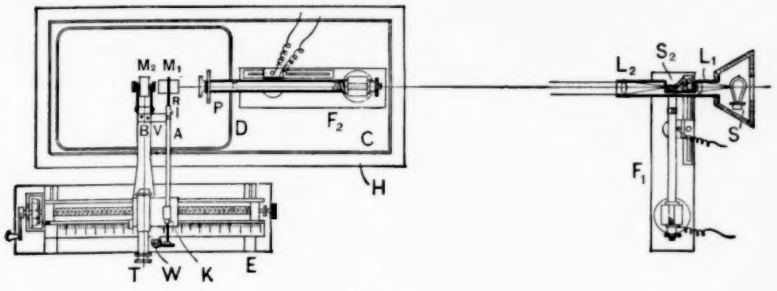

Fig. 1.

near its upper end a small hole so placed that when the fork is at rest these holes shall lie one directly behind the other in a horizontal line. F_2 is the fork which generates the ripples by means of the plate of glass (P) 2.5 by 9.7 cm. L_1 and L_2 are two small double-convex lenses. D is the vessel for holding the liquid to be experimented upon; it is a porcelain developing tray 2.3 by 29 by 36 centimeters. E is a dividing engine furnished with a screw whose pitch is 1.0328 millimeters. M_1 and M_2 are silver-on-glass mirrors. T is a telescope. To the carriage of the dividing engine is rigidly fastened a wooden arm (B) and a brass tube (A); the tube is parallel to the wooden arm and is fastened to it by means of V. To the end of B is fastened the mirror M_2 by means of brass mountings that allow it to be turned and clamped in any position desired. Through the tube A passes a brass rod (R) to which is fastened the mirror M_1. This allows M_1 to be rotated about the axis of the tube, and it is clamped in the necessary position by means of the screws I and K. W is an iron weight hung from a rod fastened to the carriage, and is intended to counterbalance the rod, mirrors, etc.

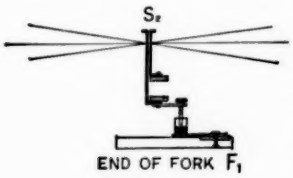

Fig. 2.

To protect the liquid as well as possible from dust and other impurities in the air, the tray D and the fork F_2 were placed in the galvanized iron box (C) which was closed by a tightly fitting lid provided with a window of good, *plane, parallel* plate glass, through which all readings were taken.

The fork F_1 is an ordinary electrically driven tuning fork pro-

vided with a mercury platinum contact maker. F_2 is also an electrically driven tuning fork. It is tuned in unison with F_1, and is driven by it by means of a shunt circuit.

By means of L_1 the light from S is focused on a point midway between the two holes in S_2; it then passes through L_2 which renders it parallel, then it is reflected by M_1 to the surface of the liquid, from this it is reflected to M_2, and from M_2 into the telescope T. If now the forks are started and the telescope is focused for the light reflected from the crests of the waves, the entire field of the telescope, which is dimly illuminated, will be crossed by a series of parallel bright lines which correspond to the crests of the waves. These lines appear stationary, and the distance between them is approximately one-half the wave-length, since each wave is seen in two positions, once when the prongs of F_1 pass through their positions of equilibrium and are approaching one another; and again when they are separating. The crests seen in one position do not lie exactly midway between those seen in the other.

The plan of operation is to set the spider line of the telescope on one of these lines, read the position of the carriage, then move it along several centimeters, and set on another line of the same series. The difference of these two readings divided by the number of waves passed over gives the wave-length.

SOURCES OF ERROR AND ADJUSTMENTS.

The first objection that presents itself is that, as the carriage is moved, the mirrors will not remain parallel to their original positions. Owing to the length of the arms supporting the mirrors (30 cm.) and to the distance of the liquid below the mirrors (45 cm.), any slight displacement of the mirrors will produce a great error in the determination of the wave-length. These errors may be introduced either by irregularities in the ways of the engine, or by a motion of the arms with respect to the carriage. The latter is simply a matter of rigid connections and can be easily remedied, so we shall first consider the former.

To test the accuracy of the ways, the rotation of the carriage was tested about three rectangular axes—one along the screw, another horizontal and perpendicular to the screw, and the third ver-

tical. To do this, a mirror was mounted on the carriage perpendicular to the length of the screw, and a telescope (placed about ten feet from it) was focused on the image in the mirror of a vertical scale fixed to the telescope. If the carriage rotates about a horizontal axis perpendicular to the screw, the image of the scale will be displaced vertically with respect to the spider lines of the telescope. The carriage was moved several times over the entire length of the ways, but no displacement of the scale was noticed, except what might be due to changing my position with respect to the stand on which the telescope was supported.

The rotations about the other axes were tested in a similar manner and with the same result. This proved that the irregularities of the ways are at most very slight.

To test the rigidity of the arms and mirror clamps, the carriage was screwed along until the spider line in the telescope on the carriage coincided with a well defined crest. It was then slipped back, leaving the nut in place on the screw, and again moved up by hand to the nut. The line again coincided with the crest. This proved that moving the carriage by hand produced no effect on the relative positions of the carriage and its appurtenances. The carriage was then again slid from the nut, and, while held with one hand it was struck several sharp, horizontal taps with a block of soft wood. It was then moved back against the nut. If the telescope and mirrors are clamped tightly the line again coincides with the crest. This proves that slight jars of the screw will introduce no error into the results if the telescope and mirrors are well clamped.

Having settled this point, the other adjustments, which will now be described, were made; and then the ways were again tested by a method which will be described further on, and which is much more delicate than the one first used.

Leaving aside the accuracy of the screw and the motion of the carriage, there are six fundamental adjustments to be made.

1. The carbon filament and the axes of the lenses L_1 and L_2 must lie in a straight line.

2. The beam of light must be composed of parallel light and

3. It must be horizontal before reflection by M_1;

4. The ways of the engine must be horizontal and at such a height that the beam of parallel light may strike the mirror M_1, and

5. They must be parallel to the beam of light ;

6. The fork F_2 must be parallel to the ways of the engine. This is easily adjusted by means of a plumb line hung from the rod R.

1. The lenses L_1 and L_2 are placed in a tin tube against diaphragms perpendicular to the axis of the tube. This makes their axes practically coincide. The lamp S is then adjusted so that the light (from the filament that is to be used) after passing through L_1 covers the centre of L_2. Then the filament and the axes of L_1 and L_2 are in a straight line.

2. The lense L_1 is moved until the light is focused on S_2 and then L_2 is moved until on reflecting the beam of light back along its original path, it appears to converge to the hole in S_2, through which it originally came. The position of L_2 is then adjusted until the light is as nearly parallel as one can detect with a small telescope focused for infinity.

3. The entire system L_2, L_1 and S, is then inclined by leveling screws until the beam of light is as nearly horizontal as can be detected by means of a water level two meters long.

4. The engine is placed on a slab of slate mounted on leveling screws and at the desired height. The horizontality of the ways is tested by means of a delicate spirit level.

5. If two strips of paper are pasted on M_1 so that their edges touch opposite sides of the part of M_1 that is covered by the beam of parallel light; and if the engine is moved until the light remains on this spot while the carriage is moved over the entire length of the ways (about 50 cm.) we may be sure that the ways are quite approximately parallel to the beam of parallel light.

If water is now placed in the tray, if the mirrors and the telescope are turned so that the light is thrown down the tube of the telescope, and if cross-threads are stretched across L_2, a dark cross will be seen in the field of the telescope when it is focused for a point about one meter below the surface of the water (which is the focal length used throughout this work). The cross-threads in the telescope are made to coincide with this cross, and the carriage is moved along the ways.

If the ways are accurate, and if the other adjustments have been well made, these crosses will coincide throughout the length of the

screw. But, owing to slight irregularities in the ways, it is found that the crosses are relatively very slightly shifted, though from scale reading 15 to scale reading 29 they remained coincident, except from 22.6 to 24.3 where one bearing of the carriage rests upon a slight dent in one of the ways. Hence, all settings were taken between 16.5 and 22, and between 25 and 27. In these regions I could detect not the slightest shift; so I think the readings are entirely free from any error due to inaccuracies in the ways. Any error in the other adjustments also causes a shift in the positions of the crosses, but this is uniform throughout the length of the ways, and so can easily be distinguished from the nonuniform displacement due to faults in the ways.

Extraneous Disturbances.

Having disposed of these difficulties we come to another that gave great trouble. This is the disturbance of the liquid by extraneous vibrations. After trying many plans I finally placed the box containing the tray of water and the fork F_2 upon an iron slab (H) weighing almost 150 pounds, and supported by long steel springs hung from a joist whose ends rested in the walls of the building but which was free from the floor above. It was hung so that when weighted for work it was about three or four centimeters above the top of a brick pier, and the intermediate space between it and the pier was loosely filled with cotton batting. If the cotton is packed neither too tightly nor too loosely, this arrangement cuts out most extraneous disturbances. The dividing engine was supported on this pier. The fork which generated the ripples, and the vessel of water were each supported on pieces of rubber tubing so that the water might not be disturbed by any slight vibration of the stand of the fork.

(*To be concluded.*)

MINOR CONTRIBUTION.

A NEW METHOD OF DETERMINING THE SPECIFIC HEATS OF LIQUIDS.

BY ROBERT L. LITCH.

THE following methods have been used for determining the specific heats of liquids: (1). Method of cooling. (2). Method of mixtures. (3). [1] Method of Thomson. (4). Electrical methods as proposed by Jamin[2] and Anmary, Weber, Pfaundler, Dieterici.[3]

The errors to which these methods are liable are too well known to need comment. I propose in this paper to outline a new method which shall involve not only the virtues lying in the so-called law of Joule but also the principle which is the foundation of the "Waterman[4] Calorimeter for the method of mixtures," viz., the maintaining of the calorimeter cup at a constant temperature throughout the experiment. By this principle the errors due to the uncertainty as to what corrections must be made for "Water Equivalent" and "Radiation correction" are avoided.

I. THEORY OF THE METHOD.

Witnin two vessels, arranged one above the other, the liquid whose specific heat is desired is placed. This particular arrangement enables the operator to easily transfer liquid from one vessel to the other. The assumption is made that the temperature of the two are different, that of the upper vessel being the lower. Though theory shows it is not necessary, yet practice has made it appear advisable to make the temperature of the cooler liquid as near 0° C. as possible. The temperature of the other is that of the room.

In the lower vessel a coil of wire is also placed. A current of known strength is passed through the coil and the temperature of the liquid is, in consequence, raised. Enough liquid is dropped into the lower vessel from the upper to maintain the temperature of the liquid in the lower constant during the operation.

Then it is evident that

$$H = sm(T - T_0)$$

[1] Pogg. Ann. 142, pp. 337.
[2] Comptes Rendus, LXX., pp. 661.
[3] Wied. Ann. 33, pp. 417.
[4] Phys. Rev., Vol. IV., No. 21.

where H=heat developed

s=specific heat

m=mass of liquid dropped

$T-T_0$=difference in temperature between that of the cooler liquid and the room.

If we substitute this value in the expression for Joule's Law, we have

$$(\mathrm{I})\quad s=\frac{rc^2t}{Jm(T-T_0)}$$

where r, c, t and J have the usual meaning.

If we take two such calorimeters containing different liquids and allow the same current to pass through them it is evident that

$$(\mathrm{II})\quad \frac{s_1}{s_2}=\frac{m_2r_1(T-T_{02})}{m_1r_2(T-T_{01})}$$

where the subscripts refer to the two calorimeters.

II. Apparatus.

My apparatus is constructed in many respects on the lines of the "Waterman Calorimeter." The cooler and air-thermometer are direct copies of the corresponding parts in that apparatus. The test-tube, however, is made of very thin glass and not of silver. It passes through a cork and with it completely closes the bulb of an air-thermometer. Inside the tube is a coil of platinum wire whose ends pierce the sides of the tube and are soldered to large copper wires. These wires serve as "leads" and run up along the sides of the tube, entering the open air through the cork at the mouth of the bulb.

The other pieces of apparatus found necessary are: (1) Two thermometers, one graduated to $\frac{1}{20}$ degree, so arranged in the cooler that the liquid flows over its bulb just at the moment of passing from cooler to test tube, and another graduated to $\frac{1}{5}$ degree for measuring the room temperature; (2) a copper voltameter; (3) a D'Arsonval galvanometer for the measurement of resistance by the *fall of potential* method; (4) storage batteries; (5) miscellaneous apparatus, such as variable resistance-boxes, direct reading ammeter, keys, etc.

III. Practice of the Method.

The operations necessary are: (*a*) determination of current; (*b*) determination of the amount of liquid dropped; (*c*) measurement of time; (*d*) measurement of resistance of coil; (*e*) observation of the temperatures of the liquid in the two vessels; (*f*) substitution in the formula

$$s=\frac{c^2rt}{Jm(T-T_0)}$$

In my experiments I have used an ordinary watch. This has been possible because by using small currents the experiment could be continued from five to ten minutes, and hence, the total number of seconds being large, an error of one second would introduce no perceptible change in the results. With a spring contact key, such as I have used, it is possible to turn the current on and off so near the second intended that the sum of the errors introduced by these two operations is undoubtedly much less than a second.

The measurement of resistance is the most important of all the operations. It is necessary to measure this during the experiment itself. It is not sufficient to know it at zero degrees and its temperature coefficient, because the exact temperature of the coil cannot be determined while the operation is going on. To regard it as being the same as the liquid is erroneous, for it is easy to see that, were it so, no flow of heat could take place from the coil to the liquid. To overcome this difficulty I have made use of the *fall of potential* method for this observation. This has the virtue of being easy of application and, for the small resistances I have used, of great accuracy. For my coil of known resistance, because of its slight change resistance due to a rise in temperature, large German-silver wire (No. 16, B. W. G.) was employed. This arrangement has been found very satisfactory.

IV. Results Obtained.

Owing to various reasons I have been unable to apply my method to but one substance, viz., water. Even with this I have taken no great precaution that it should be absolutely pure and hence the results are given merely to show how the method works. Rowland's statement that the specific heat of water decreases between 4° and 30° C. is seen to be followed in these results:

T	s
18.8°	.98075
19.7	.98064
21.05	.98035
21.2	.98035

These values lie between those given by Dieterici and Velten for 20°, the former obtaining .9893, the latter .9794. The method of mixtures has invariably given for this temperature values greater than unity.

V. Second Method.

The values of J used in the experiments under the first method are those due to Rowland. They are probably the best so far obtained but notwithstanding this fact there is undoubtedly a great degree of inde-

terminateness as to the correct value of this constant and hence it is best to eliminate it, if possible, from any method of experimentation.

A glance at equation (II) reveals the fact that we require no knowledge of (1) current, (2) time, (3) mechanical equivalent. Why these three quantities disappear from the equation appears in the following considerations. The two calorimeters necessary for the practice of this method have their coils joined in series and hence the same current flows through both. The consequence is that whatever fluctuation in the current occurs, the effect in one is as great as in the other. It follows, therefore, that at every instant the current through the first is equal to that through the second and hence the ratio of the two is vigorously unity. Similar reasoning will show why the other two quantities are eliminated.

The first method requires a knowledge of all three and in addition we are compelled to make use of the average current. Trouble arises here because when set into the formula it must be squared giving a value $[1/t\Sigma i]^2$ whereas we desire $1/t\Sigma i^2$. And while it may be shown that "For small variations of the current the mean of the squares may be equated to the square of the mean," yet any approximation it may become capable of furnishing is entirely dependent on the hypothesis that the variations are small. But as it is impossible to ensure that they shall be so, any results derived on the assumption that they are must involve an error of indeterminate magnitude.

For these reasons, then, the second method appears much more satisfactory. Up to the present time I have been unable to try it, but shall attempt to do so at a later day.

In conclusion I take this opportunity to express the obligations I feel myself under to Prof. W. F. Magie and Dr. F. A. Waterman, the latter of whom has labored with me in a most kindly way giving hints and suggestions which his work with his own calorimeter has shown him would be advantageous and which have proven so in every instance.

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NEW BOOKS.

Experimental Physics. By W. A. STONE. 8vo, pp. 378. Boston, Ginn & Co., 1897.

First Principles of Natural Philosophy. By A. E. DOLBEAR. 8vo, pp. 318. Boston, Ginn & Co., 1897.

The recognition of laboratory exercises as an important factor in the teaching of elementary physics has led to the repetition of the old experience that a young and long neglected party, which at first claimed equality only with an older one, gradually tries to assume all the rights and leave nothing to the other. Mr. Stone seemingly is a partisan of the new régime, but not to such a degree that he expects a student to be able to deduce the laws of nature unaided. The elegant way in which the author makes the older "text-book" politely concede the precedence to the newcomer, but afterwards guide him along the proper road by well put questions and directions, will commend this book to any teacher using the laboratory method exclusively. Quantitative experiments are not undertaken until the student has learned by qualitative work something about that which he is going to measure. It is to be regretted that in the deduction of the laws so little use has been made of mathematical expressions as a simple means of combining several wordy rules, while on the other hand the introduction of the graphical method in its simplest form is a great step in advance.

The arrangement of the subject is quite original. Mechanics forms the sixth of the eight chapters of the book, the preceding ones being: Mensuration and Hydrostatics, Heat, Elasticity, Sound, Light. The reason for this change from the usual method lies probably in the attempt to make the subject as palatable as possible for the beginner, who often loses the taste for physics by being overworked with experiments in mechanics. It seems, however, very peculiar that the study of mass, force, work, motion, etc. (in this order!) is begun on page 245. Mr. Stone has had an experience of nearly ten years in teaching the subject and it is impossible for anyone who has no experience as to the practical working of the above method, to form a definite judgment on it; but it seems to me, that questions must come up in the very beginning of the course, the answer to which is reserved for the second half of the book.

The first six chapters are treated very satisfactorily, while this cannot be said of the chapter on electricity. The book is intended as well for

students not preparing for college as for those who do, and it will be, especially for the former class, an unsatisfactory course in physics, that teaches them nothing of static electricity, electro-magnetic induction and all the important applications of electricity: the telegraph and telephone, dynamo-electric machinery and the electric light. It must be admitted that it is very difficult, to include these subjects in a book based on the laboratory method, but they ought not to be left out in any course on physics, however unpretentious.

Mr. Stone's book is intended primarily for schools preparing for Harvard and will be of great value for such, since it contains the experiments recommended by that university as a good preparation for physics.

Professor Dolbear's "text-book" is of an entirely different type and intended for a different class of students. It was to be expected that the author of "Matter, Ether and Motion" would "direct the attention of the student from the physics of mechanism to the physics of the molecules and help him to carry the mechanical conceptions gained by the study of visible bodies to their ultimate particles." The vortex-ring is introduced at the very beginning and its vibrations play a very important part in the explanation of thermal, optical and electrical phenomena. These very interesting speculations, though it seems to me, too advanced for a beginner, justify fully the return to the old name: "Natural Philosophy."

The very conservative standpoint of the author is best shown by the exclusive use of the English units, while the C. G. S. system has not even been mentioned. In what a sharp contrast does this stand to last year's unanimous resolution, of the American Association, demanding a reform in weights and measures and further legislation looking to the early adoption of the metric system. Unfortunately there are several misleading statements and misprints in the book that require great care in its use.

The popular character of the book taken as a whole, and the fact, that its study requires hardly any mathematical preparation, will introduce it into certain schools and courses of physics, where other text-books are only a burden to the teacher and entirely too advanced for the students.

K. E. GUTHE.

Alternating Current Phenomena. By CHARLES P. STEINMETZ. 8vo, 424 pp. New York, W. J. Johnston Company, 1897.

The development of the commercial application of electricity is a most curious incident in the history of human progress. The lightning's flash, and the reverberating roll of the thunder revealed the presence of this form of energy to primeval man, while from the far north, the pale gleams of the aurora contained a prophesy of Mr. Tesla's Wireless

Transmission, and heralded all the wonders of the vacuum tube and cold light. Twenty centuries ago the Greeks baptized electricity, but seemingly were poor sponsors, caring but little for the development of their progency, for excluding a Celestial knowledge of the compass, and a few lines in the Vedas which fanciful translators strain into a reference to incandescent lighting and the telephone, there was not a single practical application of electricity until near the middle of the present century. Indeed, a majority of us of barely middle age, withal, remember that the electrical engineering courses of the best schools of our youth, consisted in an inspection of a telegraph relay, and a study of repulsion as manifested by a long haired comrade mounted upon a insulating stool. But "mutatis, mutandis," for if the 19th century be termed "The Age of Mechanics" its last two decades form the "Era of Electricity," as during this period the development of industrial electrical appliances has caused an average annual investment of $100,000,000.00. As an urban illuminant electricity is all but universal; we talk over half a continent; Puck's girdle may almost be accomplished in forty seconds; the mention of a "horse car" provokes a smile; the energy of the greatest cataract on the globe, now diverted, is directed and delivered at pleasure over a network of conductors. To what shall this phenomenal development be ascribed? Why did the germs of electrical science lie dormant for two thousand years to sprout, in two decades with such cryptogamic rapidity? How happens it that electricity in twenty years, out-strips steam engineering of two hundred years?

It is additionally curious to note the present atavistic tendency of electrical progress. Excluding the telegraph and electro-plating, all of the original attempts at industrial application employed alternating currents. Witness the Jablochoff candle; the early permanent magnet dynamos, and the pulsations of telephony; but forsooth, because the galvanic battery warped sprouting electrical intelligence into considering that only uni-directional energy was commercial, the far more flexible and docile wave manifestation was abandoned, and electricians, imprisoning themselves with communtator segments, declared alternating currents to be dangerous, unmanagable, expensive and impracticable. But thanks to just such men as the author of alternating current phenomena—men having the ability both to perceive relationships, and to intelligently express them—old limitations are disappearing, and all electrical operations are becoming reduced to scientific unity. And just here the clear sighted perceive the true reason for the singular rapidity of electrical development. All knowledge is but an empiric record of experience, but so long as any art is but a series of scattered empiricisms, progress is impossible, and it is not until reason, ascending from the deductive to the higher levels of inductive ratiocination, aided by a scien-

tific use of the imagination, lights the way from the known into the unknown, that real and substantial advancement is made. The mathematicians are the architects of the present electrical structure, and alternating current phenomena presents the latest and most complete exposition of what might be termed "Electrical Mathematics" for development is reciprocal, and as electric science would be impossible without mathematics, conversely electrical investigation has constructed for itself a special branch of symbolic expression.

In 1893 Mr. Steinmetz prepared the way for the present volume by presenting to the International Electrical Congress, a paper on the application of the "Algebra of Complex Quantities to Electrical Problems." While the title of the paper was alarming, it was shown that the use of plane algebra greatly facilitated and abridged electrical calculations, being almost essential to a comprehension of the more involved relationships of alternating currents, and upon the foundation thus established, the author has now elaborated his solutions until they embrace a consideration of nearly all the phenomena at present applying to electrical currents. Abandoning preconceived notions of electricity, as restricted to continuous currents, Mr. Steinmetz prepares the way to a study of his methods by three chapters, occupying some twenty pages of introductory matter in which electrical laws, so broadly generalized as to apply to all phases of this manifestation, are enumerated. In the next three chapters the three fundamental processes of calculation are explained, Graphical Representation by means of polar coördinates being placed first. Passing to symbolic methods the author next shows that, while the clearest mental picture of the reciprocal reactions of alternating currents may be formed by Graphical Representation, it is often practically impossible to get accurate results, owing to draughting limitations, thus indicating the superiority of symbolic calculation. This chapter, in connection with the various explanatory appendices, will place the student in a position to handle plane algebra as applied to electrical currents. As a purely mathematical treatise this portion of the work may seem restricted, yet the busy engineer will welcome the kernel stript of the quaternionic husk of the more voluminous works on higher mathematics, and the author is to be congratulated upon successfully exorcising the famous bug-bear of $\sqrt{-1}$. In the succeeding Topographic Method, is presented an ingenious graphical protrayal of the reactions of sine waves in inter-linked circuits, that avoids some of the incongruities exhibited in polar diagrams.

Having thus developed the general scheme of analysis, the fundamental characteristics of circuits: Admittance, conductance, susceptance, resistance, inductance and capacity are taken in order, both symbolically and graphically, and analytical methods applied to determine the effect

in general of the presence of these factors and their combinations. So far the volume is occupied purely with general mathematical statements, although reinforced with adequate practical illustrations, but in Chapters IX., X. and XII. the application of these principles to transmission lines (the important problem to practicing engineers), is taken in hand in an extremely valuable manner. Logically, Chapter XI., on Foucault Currents, and a portion of Chapter X., dealing with the effect of hysteresis, appear out of place, and would seem more properly found as an introduction to the subsequent sections on alternating current machinery. Completing the investigation of the properties of transmission circuits, Mr. Steinmetz, in Chapter XIII., commences the study of alternating current machinery with an extensive investigation of the transformer. The induction motor receives a fair share of attention, while synchronous motors and generators are treated in extenso. Communtator motors and reaction machinery are mentioned seemingly for the sake of completeness, as little encouragement is offered for their practical utilization. The last third of the volume is occupied with wave distortion, the effect of harmonies and a consideration of general polyphase systems, including an investigation into the required amount of conducting material. Here again some illogicality in arrangement is apparent.

Beyond question, Mr. Steinmetz has carried mathematical analysis, as applied to electricity, further than any other American engineer; so far, indeed, that time is the only criterion with which to test the work, for at present his methods extend over the confines of practice into the domain of pure theory. But the author has given the profession an edged tool, so keen withal, that a wise man may carve himself a fortune or the fool kill himself therewith, and so a reference to the aphorism regarding cutting edges is apposite.

There is a wealth of information in the book, profusely illustrated with supposititious practical examples, for which the author deserves the gratitude of all electricians; but it is so concentrated that those of us who are not mathematical ostriches are likely to have an attack of intellectual dyspepsia. To Lord Kelvin or Mr. Heaviside, alternating current phenomena would probably be only light literature, to accompany black coffee and a post-prandial cigar, but for the rank and file of electrical engineers, the food is too concentrated and should be peptonized with additional explanation. Typographically the volume is in accord with the well-known work of the publishers and the absence of errors in matter of so symbolic a character is notable, but in the diagrams it is again a pity that so much compression is employed as to make the subscript letters microscopically illegible.

Prophets are without honor, not only in their own country, but as well in their day and generation, and Mr. Steinmetz will have to wait the full-

ness of time to receive the appreciation his work unquestionably deserves, but if the present rapidity of electrical development continues, his reward will not be a posthumous one.

A. V. ABBOTT.

The Polarizing Photo-Chronograph. By ALBERT CUSHING CREHORE, PH.D. and GEORGE OWEN SQUIER, PH.D. Pp. vii+150. New York, John Wiley & Sons, 1897.

This volume consists of a reprint of the three papers upon the measurement of the velocity of projectiles made by Messrs. Crehore and Squier at the United States Artillery School, Fort Monroe, Virginia, and originally published in the *Journal of the United States Artillery*. The authors have added a brief introduction of five pages, and have published Dr. Crehore's paper describing the origin and development of his method of recording variable current curves [1] as an appendix to the volume. The placing of this paper in an appendix instead of making it the introductory number of the series may be regarded as something of a deviation from the plan of presentation outlined in the preface; since it was the first in the order of time and likewise first in the history of the development of the method. The arrangement adopted is, however, justified in the fact that the method underwent very radical changes before its application to the measurement of the motion of projectiles.

The work of Crehore and Squier affords a striking illustration of the way in which, from time to time, phenomena in physics which seem at first sight most remote from any possibility of utilitarian interest, are put to practical use. The rotation of the plane of the polarization of light in the magnetic field would surely have been considered such a phenomenon; and yet we suddenly find it utilized in the furtherance of an important art. A later communication from the same authors [2] foreshadows its application, indeed, in the much broader and more important field of high speed telegraphy.

The experiments of which the book treats are clearly described in terms which will render them intelligible and interesting both to the man of science and to the military engineer. The interest of the physicist in the memoirs consists more particularly in the possibilities which they suggest of further applications of the method to the registration of short intervals of time than in the results which have been obtained upon the flight of projectiles. To the student of the art of practical gunnery, on the other hand, the data given will, on account of their definite and accurate character, be of the greatest service.

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[1] PHYSICAL REVIEW, Vol. II, p. 122.

[2] Transactions of the American Institute of Electrical Engineers, April, 1897.

Contributions to the Analysis of the Sensations. By ERNST MACH. Translated by C. M. WILLIAMS. 8vo, pp. viii+208. Chicago, The Open Court Publishing Co., 1897. (*Received.*)

The Scientific Papers of John Couch Adams. Edited by WILLIAM GRYLLS ADAMS, with a Memoir by J. W. L. GLAISHER. pp. liv and 502. Cambridge, at the University Press, 1896. (*Received.*)